เฉลยข้อสอบ Final ชุด2:

6.7 จงเขียน $\dfrac{(x-1)(x-3)}{x(x^2+1)(x^2-4)}$ ให้อยู่ในรูปผลบวกของเศษส่วนย่อย (ไม่ต้องหาค่าตัวแปร).

$(x-2)(x+2)$

$$= \frac{A_1}{x} + \frac{A_2x+A_3}{x^2+1} + \frac{A_4}{x-2} + \frac{A_5}{x+2}$$

7.7 จงหา. $\displaystyle\int \frac{x^3-2x-2}{x^2(x^2+2x+2)}\,dx$ $\rightarrow$ $x = \dfrac{-2 \pm \sqrt{(4-4\cdot 2)}}{2}$, $(4-4\cdot2) < 0$ ✗

แยกตัวประกอบไม่ได้.

พิจารณา.

$$\frac{x^3-2x-2}{x^2(x^2+2x+2)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{Bx+C}{x^2+2x+2}$$

คูณตลอดด้วย $x^2(x^2+2x+2)$

$$\Rightarrow x^3-2x-2 = A_1\underbrace{x(x^2+2x+2)}_{x^3+2x^2+2x} + A_2(x^2+2x+2) + \underbrace{(Bx+C)(x^2)}_{=Bx^3+Cx^2}$$

เทียบสปส.

$$1x^3+0x^2-2x-2 = (A_1+B)x^3 + (2A_1+A_2+C)x^2 + (2A_1+2A_2)x + 2A_2$$

① $A_1+B = 1$

② $2A_1+A_2+C = 0$

③ $2A_1+2A_2 = -2$

④ $2A_2 = -2$

① $A_1 + B = 1 \Rightarrow B = 1 - A_1 = 1 - 0 = 1$

② $2A_1 + A_2 + C = 0 \Rightarrow C = -(2A_1 + A_2) = -(0+(-1)) = 1$

③ $2A_1 + 2A_2 = -2 \Rightarrow A_1 = -1 - A_2 = -1-(-1) = 0$

④ $2A_2 = -2 \Rightarrow A_2 = -1$

ดังนั้น. $A_1 = 0,\ A_2 = -1,\ B = 1,\ C = 1$

ดังนั้น

$$\int \frac{x^3 - 2x - 2}{x^2(x^2+2x+2)}\,dx = \int \frac{0}{x} + \frac{-1}{x^2} + \frac{x+1}{x^2+2x+2}\,dx$$

$$= -\int \frac{1}{x^2}\,dx + \int \frac{x+1}{x^2+2x+2}\,dx$$

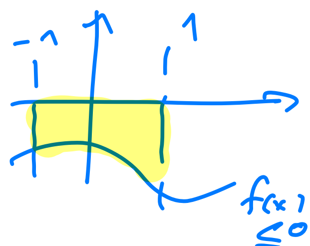
$x^2+2x+2 = u \Rightarrow du = 2x+2\,dx = 2(x+1)\,dx$

$$= \frac{1}{x} + \int \frac{(x+1)}{u} \frac{du}{2(x+1)} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{x} + \frac{1}{2}\ln|x^2+2x+2| + C$$

8.) กำหนด f เป็นฟังก์ชันที่ $f(x) \le 0$ บน $[-1,1]$

ถ้า $\left(\int f(x)\,dx = x^3 - 3x + 1\right)$



ปฏิยานุพันธ์ของ f เป็น

จาก พท. ระหว่างเส้นโค้ง $y = f(x)$ และแกน x เมื่อ $-1 \le x \le 1$.

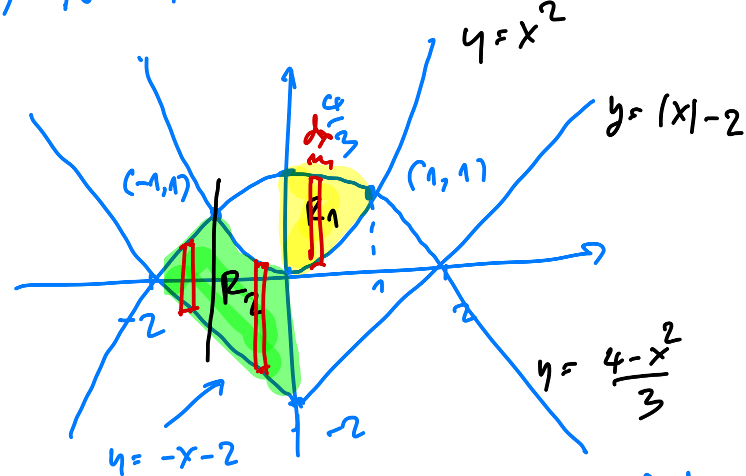
$$\Rightarrow \text{พ.} \int_{-1}^{1} f(x)\,dx = \left[x^3 - 3x + 1\right]\Big|_{x=-1}^{x=1}$$

$$\Rightarrow \left[(1^3 - 3\cdot 1 + 1) - ((-1)^3 - 3(-1) + 1)\right]$$

$$= (-1) - 3 = \underline{\underline{-4}}$$

∴ พท. $= \underline{4}$ หน่วย^2

ข.) ให้ R_1, R_2 เป็น พท. ดังรูป



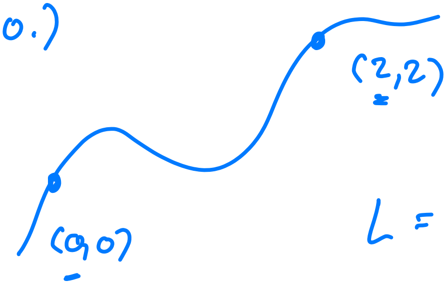
ข.1: เขียน พท. R_1 ในรูป อินทิกรัล (ไม่ต้องคำนวณ)

$$R_1 = \int_{x=0}^{x=1} \left(\frac{4-x^2}{3}\right) - x^2 \; dx$$

9.2: $R_2 = \int_{x=-2}^{x=-1} \left(\frac{4-x^2}{3}\right) - (-x-2)\, dx$

$$+ \int_{x=-1}^{x=0} x^2 - (-x-2)\, dx \qquad \boxtimes$$

10.)



$y = \sqrt{4x - x^2}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{\cancel{2}} (4x - x^2)^{-\frac{1}{2}} (\cancel{4}^{\,2} - \cancel{2}x)$

$$L = \int_{x=0}^{x=2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{x=0}^{x=2} \sqrt{1 + \left(\frac{2-x}{\sqrt{4x-x^2}}\right)^2}\, dx$$

$$= \int_{x=0}^{x=2} \sqrt{1 + \frac{(2-x)^2}{4x-x^2}}\, dx$$

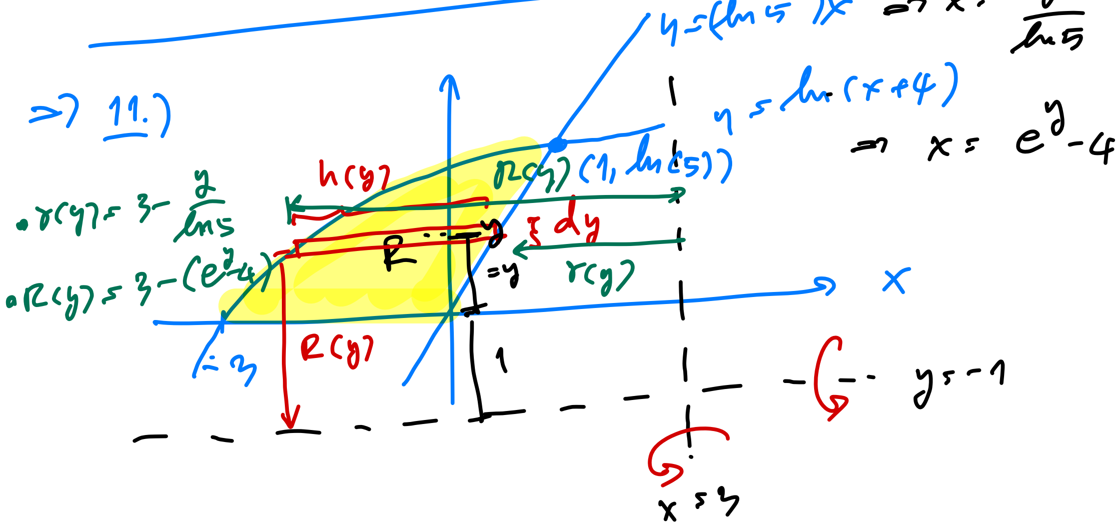
$$= \int_{x=0}^{x=2} \sqrt{\frac{\cancel{4x} - \cancel{x^2} + 4 - \cancel{4x} + \cancel{x^2}}{4x-x^2}}\, dx = 2\int_{x=0}^{x=2} \sqrt{\frac{1}{\underbrace{4x-x^2}}}\, dx$$

$$4 - \underbrace{(x^2 - 4x + 4)}_{(x-2)^2}$$

$$= 2\left(\operatorname{arcsin}\left(\frac{x}{2} - 1\right)\right)\Big|_{x=0}^{x=2} = \ldots \quad \boxtimes$$

$$= 2\left[\underbrace{\arctan(0)}_{0} - \underbrace{\arctan(-1)}_{\left(-\frac{\pi}{2}\right)}\right] = 2\cdot\frac{\pi}{2} = \pi \quad \blacksquare$$

$\Rightarrow$ **11.)**



11.1. หมุน R รอบ $y=-1$ โดยวิธี shell. ‖ //แกนหมุน.

$$V = \int_{y=0}^{y=\ln 5} 2\pi\, R(y)\cdot h(y)\, dy$$

- $R(y) = y + 1$
- $h(y) = \left(\dfrac{y}{\ln 5}\right) - \left(e^{y} - 4\right)$

$$= \int_{y=0}^{y=\ln 5} 2\pi (y+1)\left[\left(\frac{y}{\ln 5}\right) - \left(e^{y} - 4\right)\right] dy \quad \blacksquare$$

11.2: หมุน R รอบ $x=3$ วิธี Disk:

$$V = \int_{y=0}^{y=\ln 5} \pi\left(R(y)^2 - r(y)^2\right) dy$$

- $r(y) = 3 - \dfrac{y}{\ln 5}$
- $R(y) = 3 - (e^{y} - 4)$

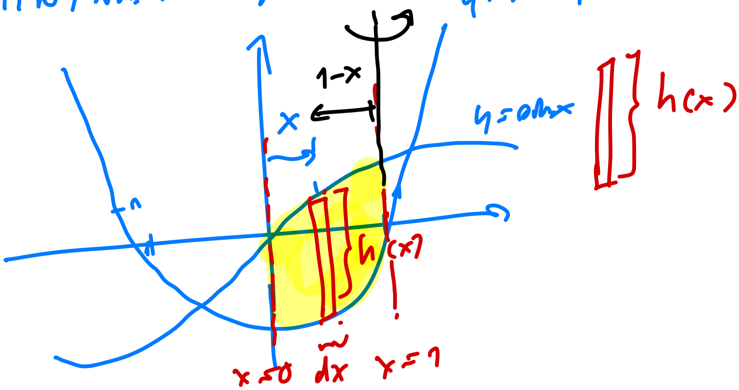
$$V = \int_{y=0}^{y=\ln 5} \pi\left(\left(3-(e^{y}-4)\right)^2 - \left(3-\left(\frac{y}{\ln 5}\right)\right)^2\right) dy$$

12.) โจทย์

$$V = \int_{0}^{1} 2\pi\,\underbrace{(1-x)}_{R(x)}\,\underbrace{(\sin x - (x^2-1))}_{h(x)}\,dx$$

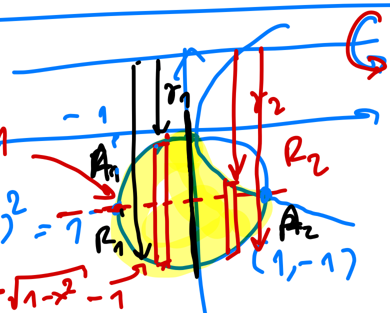
ใช้ shell

12.1.) วาดกราฟ พท. ที่โจทย์ให้:



12.2. หมุนรอบเส้นตรง: $x = 1$.

13:



$\Rightarrow y = +\sqrt{1-x^2} - 1$

$x^2 + (y+1)^2 = 1$

$y = -\sqrt{1-x^2} - 1$

$x = y^2 \Rightarrow y = -\sqrt{x}$

$x = 1$

$x = 0$

Disk: $V = \int_{x=-1}^{x=0} \pi\left(R_1(x)^2 - r_1(x)^2\right)dx + \int_{x=0} \pi\left(R_2(x)^2 - r_2(x)^2\right)dx$

- $R_1(x) = 1 - \left(-\sqrt{1-x^2} - 1\right)$
- $r_1(x) = 1 - \left(+\sqrt{1-x^2} - 1\right)$

$R_2(x) = 1 - \left(-\sqrt{1-x^2} - 1\right)$

$r_2(x) = 1 - \left(-\sqrt{x}\right)$

$$\Rightarrow V = \int_{x=-1}^{x=0} \pi \Big[\big(1 - \underbrace{(-\sqrt{1-x^2} - 1)}_{R_1}\big)^2 - \big(\underbrace{1 - (+\sqrt{1-x^2} - 1)}_{r_1}\big)^2 \Big] \, dx$$

$$+ \int_{x=0}^{x=1} \pi \Big[\big(\underbrace{1 - (-\sqrt{1-x^2} - 1)}_{R_2}\big)^2 - \big(\underbrace{1 - (-\sqrt{x})}_{r_2}\big)^2 \Big] \, dx$$

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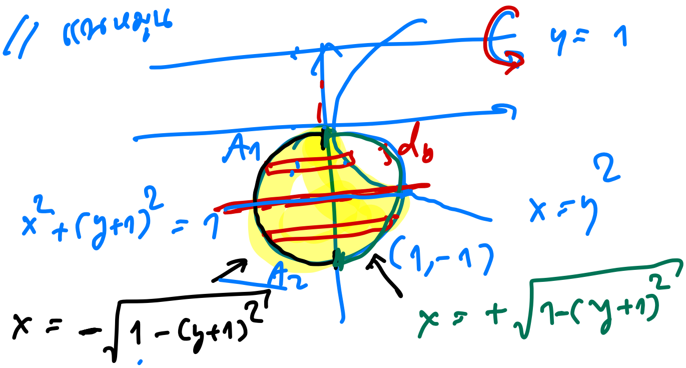
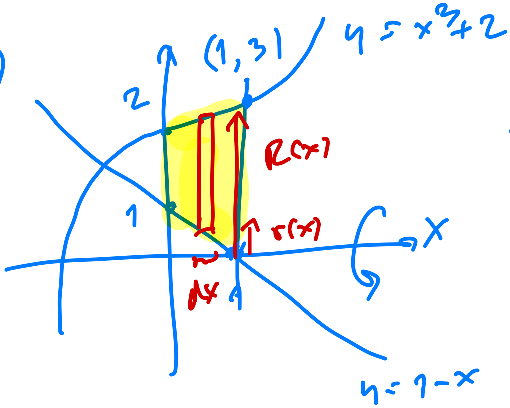
shell: ∥ // [illegible]



$$V = \int_{y=-1}^{y=0} \pi (1-y)\left(y^2 - \left(-\sqrt{1-(y+1)^2}\right)\right) dy$$

$$+ \int_{y=-2}^{y=-1} \pi (1-y)\left[+\sqrt{1-(y+1)^2} - \left(-\sqrt{1-(y+1)^2}\right)\right] dy$$

1(c.)



$$V = \int_{x=0}^{x=1} \pi \left(R(x)^2 - r(x)^2 \right) dx$$

- $R(x) = x^3 + 2$
- $r(x) = 1 - x$

$$\Rightarrow \quad V = \int_{x=0}^{x=1} \pi \left[(x^3+2)^2 - (1-x)^2 \right] dx$$

$$= \pi \int_{x=0}^{x=1} (x^6 + 4x^3 + 4) - (1 - 2x + x^2)\, dx$$

$$= \pi \left[\frac{x^7}{7} + \frac{4x^4}{4} - \frac{x^3}{3} + \frac{2x^2}{2} - x \right] \Bigg|_{x=0}^{x=1}$$

$$= \pi \left[\frac{1}{7} + 1 - \frac{1}{3} + \cancel{1} - \cancel{1} \right] = \frac{21-4}{21}\pi = \frac{17}{21}\pi$$