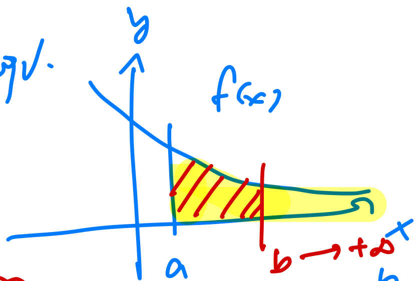
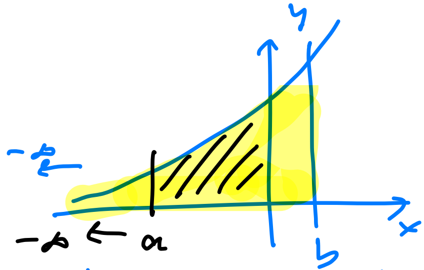
→ อินทิกรัลไม่ตรงแบบ: 2 รูปแบบ

$\boxed{1}$: ขอบเขตเป็น $\pm\infty$

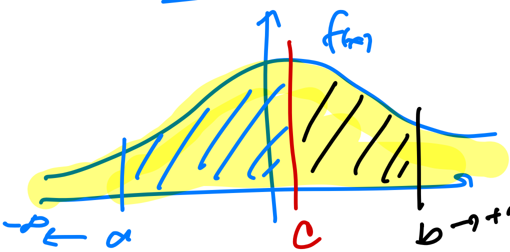
อยู่ในรูป:



$$\int_a^{+\infty} f(x)\,dx = \lim_{b\to+\infty}\int_a^b f(x)\,dx$$



$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

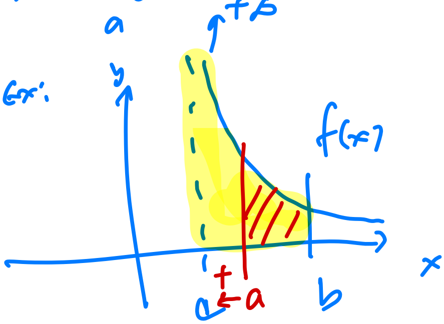


$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_a^{+\infty} f(x)\,dx$$

$$= \lim_{a\to-\infty}\int_a^c f(x)\,dx + \lim_{b\to+\infty}\int_a^b f(x)\,dx$$

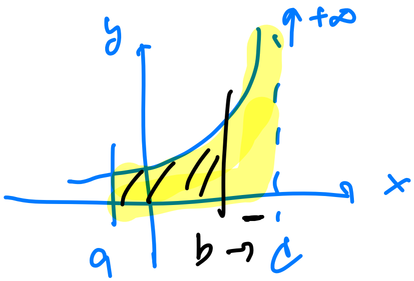
$\boxed{2}$: $\int_a^b f(x)\,dx$ แล้วมี $c\in[a,b]$ ที่ $f(c)=\pm\infty$

Ex:



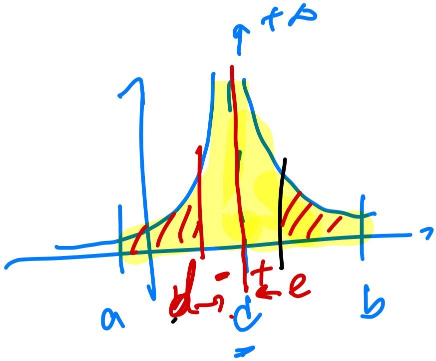
$$\int_c^b f(x)\,dx \text{ โดยที่ } f(c)=+\infty$$

$$= \lim_{a\to c^+}\int_a^b f(x)\,dx$$



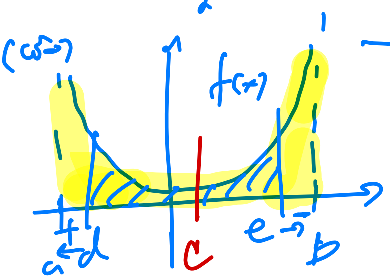
$\int_a^c f(x)\,dx$ โดยที่ $f(c) = +\infty$

$= \lim_{b \to c^-} \int_a^b f(x)\,dx$



$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$= \lim_{d \to c^-} \int_a^d f(x)\,dx + \lim_{e \to c^+} \int_e^b f(x)\,dx$



$\int_a^b f(x)\,dx$, $f(a) = +\infty$, $f(b) = +\infty$

$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

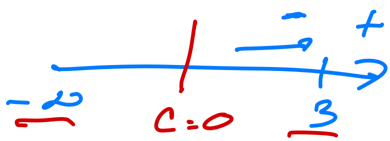
$= \lim_{d \to a^+} \int_d^c f(x)\,dx + \lim_{e \to b^-} \int_c^e f(x)\,dx$

แบบที่ $\boxed{3}$: แบบผสม.

$f(x) = \pm\infty$ เมื่อ $x = 3, 4$

ช่วง. $(-\infty, 3)$

Ex: $\int_{-\infty}^{3} \frac{1}{(x-3)(x-4)}\,dx$



จงพิ. $\displaystyle\int_{-\infty}^{3} \frac{1}{(x-3)(x-4)}\,dx = \int_{-\infty}^{0} \frac{1}{(x-3)(x-4)}\,dx + \int_{0}^{3} \frac{1}{(x-3)(x-4)}\,dx$

$$= \lim_{a \to -\infty} \int_{a}^{0} \frac{1}{(x-3)(x-4)}\,dx + \lim_{b \to 3^-} \int_{0}^{b} \frac{1}{(x-3)(x-4)}\,dx$$

$$\left[\ \frac{1}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4} \quad \text{หา } A, B\ \right]$$

แบบฝึกหัดท้ายบท Final ชุด 2 ;

1.) จงหาอินทิกรัลต่อไปนี้.

1.1.) $\displaystyle\int \frac{2}{\sqrt{x}} + \frac{5}{x^2} + \frac{6x}{x^2+1} + \frac{1}{x}\,dx$

$$= \frac{2x^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)} + \frac{5x^{-1}}{-1} + \int \frac{6x}{x^2+1}\,dx + \ln|x| + C$$

$u = x^2+1$

$\dfrac{du}{dx} = 2x \Rightarrow dx = \dfrac{du}{2x}$

$$\int \frac{\cancel{6x}\,3}{u} \cdot \frac{du}{\cancel{2x}}$$

$$= 3\ln|u| = 3\ln|x^2+1| + C$$

1.2.) $\int (2x+18)^5 + (2+18x)^{1/5}\, dx$

$$= \int (2x+18)^5\, dx + \int (2+18x)^{\frac{1}{5}}\, dx$$

$\Rightarrow u = 2x+18$

$du = 2dx$

$dx = \frac{du}{2}$

$u = 2+18x$

$du = 18\,dx$

$dx = \frac{du}{18}$

$$\Rightarrow \int u^5 \frac{du}{2} \qquad \int u^{\frac{1}{5}} \frac{du}{18}$$

$$\frac{u^6}{6\cdot 2} + C \qquad \frac{u^{\frac{1}{5}+1}}{18\left(\frac{1}{5}+1\right)} + C$$

แทน u กลับ

$$= \frac{(2x+18)^6}{12} + 5\cdot\frac{(2+18x)^{\frac{6}{5}}}{18\cdot 6} + C$$

1.3) $\int e^{\sin x} \cos x - 2^{\cos x} \sin x\, dx$

$$= \int e^{\overbrace{\sin x}^{=u}} \cos x\, dx - \int 2^{\overbrace{\cos x}^{=v}} \sin x\, dx$$

$u = \sin x$

$du = \cos x\, dx$

$dx = \frac{du}{\cos x}$

$v = \cos x$

$dv = (-\sin x)\, dx$

$dx = \frac{dv}{(-\sin x)}$

$$= \int e^{u} \cancel{\cos x} \frac{du}{\cancel{\cos x}} + \int 2^{v} \cancel{\sin x} \frac{dv}{-\cancel{\sin x}}$$

$$= e^{u} + \frac{2^{v}}{\ln 2} + C$$

แทนกลับ $= e^{\sin x} + \frac{2^{\cos x}}{\ln 2} + C$ ∎

1.4.) $\int \left[(1-\tan x)^{9/2} + \cot x\right] \sec^2 x \, dx$

$$= \int \left[(1-\tan x)^{\frac{9}{2}} + \frac{1}{\tan x}\right] \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$dx = \frac{du}{\sec^2 x}$

แทน u

$$= \int \left[(1-\underbrace{\tan x}_{u})^{\frac{9}{2}} + \frac{1}{\underbrace{\tan x}_{u}}\right] \cancel{\sec^2 x} \frac{du}{\cancel{\sec^2 x}}$$

$$= \int (\underbrace{1-u}_{=v})^{\frac{9}{2}} du + \int \frac{1}{u} du$$

ระวัง! $\Rightarrow dv = -du$

$$= -\frac{(1-u)^{\frac{9}{2}+1}}{\left(\frac{9}{2}+1\right)} + \ln|u| + C$$

yerine $u = \tan x$

$$= \frac{-2(1-\tan x)^{\frac{11}{2}}}{11} + \ln|\tan x| + C \quad \boxtimes$$

1.5.) $$\int \frac{4x}{\sqrt{1-(1-x^2)^2}}\,dx$$

$$\begin{array}{|l} u = 1-x^2 \\ du = -2x\,dx \\ dx = \dfrac{du}{-2x} \end{array}$$

$$= \int \frac{\cancel{4x}\,2}{\sqrt{1-u^2}} \frac{du}{\cancel{-2x}} = -2\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= -2\arcsin(u) + C \overset{\text{yerine koy}}{=} -2\arcsin(1-x^2) + C \quad \boxtimes$$

1.6). $$\int x^4 \operatorname{cosec}(\underbrace{5x^5+5}_{=u}) \cot(\underbrace{5x^5+5}_{=u})\,dx$$

$$\begin{array}{|l} u = 5x^5+5 \\ du = 25x^4\,dx \\ dx = \dfrac{du}{25x^4} \end{array}$$

yerine u
$$= \int \cancel{x^4} \operatorname{cosec}(u)\cot(u)\frac{du}{25\cancel{x^4}}$$

$$= \frac{1}{25}(-\operatorname{cosec}(u)) = -\frac{1}{25}\operatorname{cosec}(5x^5+5) + C \quad \boxtimes$$

2.) παραδείγματα Ολοκληρώματα by parts.

2.1.) $$\int \frac{\ln(x^2+1)}{2x^2}\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = \ln(x^2+1)$

$$\int dv = \int \frac{dx}{2x^2} \Rightarrow v = -\frac{1}{2x}$$

by parts

$$= \ln(x^2+1)\left(-\frac{1}{2x}\right) - \int \left(-\frac{1}{\cancel{2x}}\right) \underbrace{d\ln(x^2+1)}_{=\frac{1}{(x^2+1)}\cdot \cancel{2x}\,dx}$$

$$= -\frac{\ln(x^2+1)}{2x} + \int \frac{1}{(x^2+1)}\,dx$$

$$= -\frac{\ln(x^2+1)}{2x} + \arctan(x^2+1) + \underline{\underline{C}}$$

2.2.) $$\int \underbrace{(x^3+1)}_{=u}\underbrace{x^2 e^{x^3}\,dx}_{=dv}$$

$u = x^3+1$

$$\int dv = \int x^2 e^{x^3}\,dx \quad \rightarrow \quad w = x^3,\ dw = 3x^2\,dx$$

$$v = \frac{e^{x^3}}{3}$$

by parts

$$= (x^3+1)\left(\frac{e^{x^3}}{3}\right) - \int \frac{e^{x^3}}{\cancel{3}}\underbrace{d(x^3+1)}_{=\cancel{3}x^2\,dx} = \frac{e^{x^3}}{3} + C$$

$$= (x^3+1)\left(\frac{e^{x^3}}{3}\right) - \frac{e^{x^3}}{3} + C$$

3.) จงหาอินทิกรัล. $\int \sin^3(2\pi x)\,dx$

$u = \cos(2\pi x)$

$\frac{du}{dx} = -\sin(2\pi x)(2\pi)$

$dx = \frac{du}{-2\pi \sin(2\pi x)}$

$$= \int \underbrace{\sin^2(2\pi x)}_{(1-\underbrace{\cos^2(2\pi x)}_{=u^2})} \cdot \cancel{\sin(2\pi x)}\, \frac{du}{-2\pi\, \cancel{\sin(2\pi x)}}$$

$$= \frac{-1}{2\pi}\int (1-u^2)\,du = \frac{-1}{2\pi}\left[u - \frac{u^3}{3}\right] + C$$

แทนค่ากลับ.

$$= -\frac{1}{2\pi}\left[\cos(2\pi x) - \frac{\cos^3(2\pi x)}{3}\right] + C$$

4.) จงหา. $\int \cos(x)\cos(2x)\cos(3x)\,dx$

$= \frac{1}{2}\left[\cos(-1x) + \cos(3x)\right]$

$\cos(mx)\cdot\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$

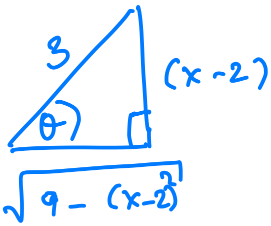
$$= \frac{1}{2}\int \cos(-x)\cos(3x) + \cos(3x)\cos(3x)\,dx$$

$$= \frac{1}{2}\int \frac{1}{2}\left[\cos(\underbrace{-4x}_{u}) + \cos(2x)\right] + \frac{1}{2}\left[\underbrace{\cos(0x)}_{=1} + \cos(6x)\right]dx$$

(เปลี่ยนกลับตัวแปร.)

$$= \frac{1}{4}\left[\frac{\sin(-4x)}{-4} + \frac{\sin(2x)}{2} + x + \frac{\sin(6x)}{6}\right] + C \quad \boxtimes$$

5.) หาค่า อินทิกรัล. $\displaystyle\int \frac{x}{\sqrt{9-(x-2)^2}}\,dx$ โดยการแทนค่า fn ตรีโกณมิติ.



- $(x-2) = 3\sin\theta \Rightarrow x = 3\sin\theta + 2 \Rightarrow dx = 3\cos\theta\, d\theta$
- $\sqrt{9-(x-2)^2} = 3\cos\theta$

เปลี่ยนตัวแปร $x \to \theta$ $\Rightarrow$

$$\int \frac{(3\sin\theta + 2)}{\cancel{3\cos\theta}}\,\cancel{3\cos\theta}\,d\theta$$

$$= -3\cos\theta + 2\theta$$

$$= -3\cdot\frac{\sqrt{9-(x-2)^2}}{3} + 2\arcsin\left(\frac{x-2}{3}\right) + C \quad \boxtimes$$