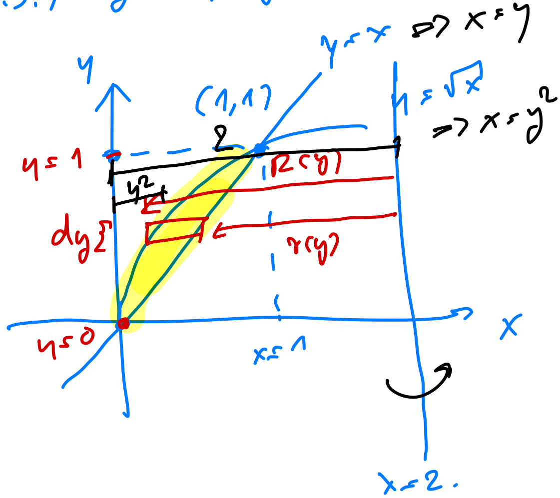
⇒ การหาปริมาตรจากการหมุน:

การบ้าน: แบบฝึกหัด 3.9 ข้อ 2.

2.3.1 $y = x$, $y = \sqrt{x}$ หมุนรอบเส้นตรง $x = 2$



⇒ จาก $y = x$ แทนใน $y = \sqrt{x}$

จะได้ $x = \sqrt{x} \Rightarrow x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

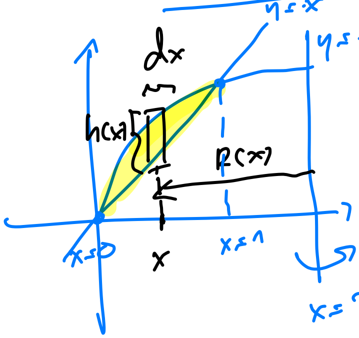
จุดตัด $x = 0$, $x = 1$

- $R(y) = 2 - y^2$
- $r(y) = 2 - y$

วิธี 1: Disk method: $[\,]$ ตั้ง $\perp$ กับ แกนหมุน.

$$V = \int_{y=0}^{y=1} \pi\left(R(y)^2 - r(y)^2\right) dy$$

แทน
$$\boxed{V = \int_{y=0}^{y=1} \pi\left[(2-y^2)^2 - (2-y)^2\right] dy.}$$ —— 1



วิธี 2: Shell Method: $[\,]$ วาง // กับ แกนหมุน

$$V = \int_{x=0}^{x=1} 2\pi R(x) \cdot h(x)\, dx$$

- $R(x) = 2 - x$
- $h(x) = \sqrt{x} - x$

$$\boxed{V = \int_{x=0}^{x=1} 2\pi (2-x)(\sqrt{x} - x)\, dx}$$ 2

Απάντηση:

$$\boxed{V = \int_{y=0}^{y=1} \pi\left[(2-y^2)^2 - (2-y)^2\right] dy.} \quad \text{—— ①}$$

$$= \int_{y=0}^{y=1} \pi\left[\underbrace{(\cancel{4}-4y^2+y^4) - (\cancel{4}-4y+y^2)}_{y^4-5y^2+4y}\right] dy$$

$$= \pi\left(\frac{y^5}{5} - \frac{5y^3}{3} + \frac{4y^2}{2}\right)\Bigg|_{y=0}^{y=1} = \overset{(3-25+30)}{\pi\left(\frac{1}{5} - \frac{5}{3} + 2\right)} = \frac{8\pi}{15} \quad \blacksquare$$

$$\boxed{V = \int_{x=0}^{x=1} 2\pi(2-x)(\sqrt{x}-x)\,dx} \quad \text{—— ②}$$

$$= 2\pi\int_{x=0}^{x=1} 2\sqrt{x} - 2x - x\sqrt{x} + x^2 \, dx = 2\pi\int_{x=0}^{x=1} 2x^{\frac{1}{2}} - 2x - x^{\frac{3}{2}} + x^2 dx$$

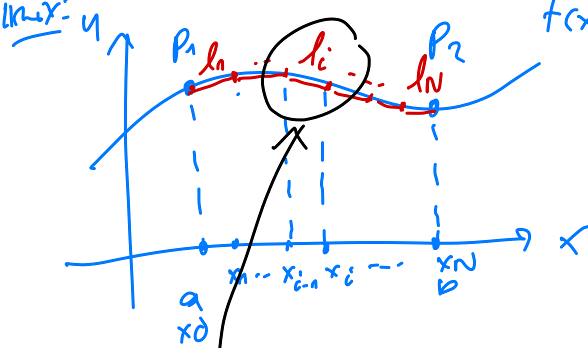
$$= 2\pi\left(\frac{2\cdot 2x^{\frac{3}{2}}}{3} - \frac{\cancel{2}x^2}{\cancel{2}} - \frac{2x^{\frac{5}{2}}}{5} + \frac{x^3}{3}\right)\Bigg|_{x=0}^{x=1}$$

$$= 2\pi\left[\left(\frac{4}{3} - 1 - \frac{2}{5} + \frac{1}{3}\right) - 0\right] = 2\pi\left(\frac{20-15-6+5}{15}\right)$$

$$= \frac{8\pi}{15} \quad \blacksquare \qquad \checkmark$$

⇒ ความยาวส่วนโค้ง:

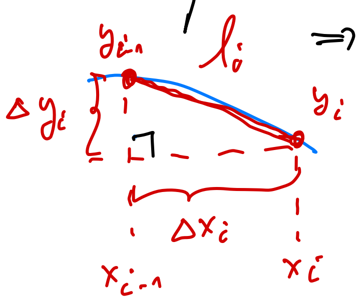
แกน x:



$$L_{P_1P_2} \approx \sum_{i=1}^{N} l_i$$

$$L_{P_1P_2} = \lim_{\substack{N\to\infty \\ (\Delta x \to 0)}} \sum_{i=1}^{N} \sqrt{1+\left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i \qquad \frac{\Delta y_i}{\Delta x_i} \Rightarrow \frac{dy}{dx}$$

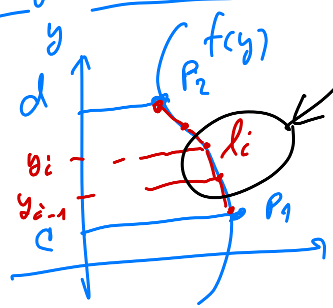
cal. ⇒ $$\boxed{L_{P_1P_2} = \int_{x=a}^{x=b} \sqrt{1+\left(\frac{df(x)}{dx}\right)^2}\,dx}$$ —(∗∗)

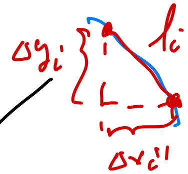


$$\Rightarrow l_i = \sqrt{\Delta x_i^2 + \Delta y_i^2}$$

$$\boxed{l_i = \sqrt{1+\left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i}$$

แกน y : ให้ทำแบบเดียวกัน





$$l_i = \sqrt{\Delta y_i^2 + \Delta x_i^2}$$

$$= \sqrt{1+\left(\frac{\Delta x_i}{\Delta y_i}\right)^2}\,\Delta y_i$$

$$\Rightarrow L_{P_1P_2} \approx \sum_{i=1}^{N} l_i = \sum_{i=1}^{N} \sqrt{1+\left(\frac{\Delta x_i}{\Delta y_i}\right)^2}\,\Delta y_i$$

ใส่ lim ⇒ $$L_{P_1P_2} = \lim_{\substack{N\to\infty \\ (\Delta y \to 0)}} \sum_{i=1}^{N} \sqrt{1+\left(\frac{\Delta x_i}{\Delta y_i}\right)^2}\,\Delta y_i$$

cal. ⇒ $$\boxed{L_{P_1P_2} = \int_{y=c}^{y=d} \sqrt{1+\left(\frac{df(y)}{dy}\right)^2}\,dy}$$

Ex: จงหาความยาวส่วนโค้งของ $y = \underbrace{(4 - x^{\frac{2}{3}})^{\frac{3}{2}}}_{f(x)}$ จาก $x=1$ ถึง $x=8$

$$L = \int_{x=1}^{x=8} \sqrt{1 + \left(\frac{df(x)}{dx}\right)^2}\, dx$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left((4 - x^{\frac{2}{3}})^{\frac{3}{2}}\right)$$

$$= \cancel{\tfrac{3}{2}}(4 - x^{\frac{2}{3}})^{\frac{1}{2}}\left(-\cancel{\tfrac{2}{3}}x^{-\frac{1}{3}}\right)$$

$$= \frac{-(4 - x^{\frac{2}{3}})^{\frac{1}{2}}}{x^{\frac{1}{3}}}$$

$$= \int_{x=1}^{x=8} \sqrt{1 + \left(\frac{-(4 - x^{\frac{2}{3}})^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^2}\, dx$$

$$= \int_{x=1}^{x=8} \sqrt{1 + \frac{(4 - x^{\frac{2}{3}})}{x^{\frac{2}{3}}}}\, dx$$

$$= \int_{x=1}^{x=8} \sqrt{\cancel{1+} 4 \cdot x^{-\frac{2}{3}} \cancel{-1}}\, dx = 2\int_{x=1}^{x=8} \underbrace{(x^{-\frac{2}{3}})^{\frac{1}{2}}}_{x^{-\frac{1}{3}}}\, dx$$

$$= 2\left(\frac{x^{-\frac{1}{3}+1}}{(-\frac{1}{3}+1)}\right)\Bigg|_{x=1}^{x=8} = 3x^{\frac{2}{3}}\Bigg|_{x=1}^{x=8} = 12 - 3 = 9$$

Ex: $x = \overbrace{(y-1)}^{f(y)}{}^{\frac{3}{2}}$ จาก $y=1$ ถึง $y=6$.

$$\Rightarrow L = \int_{y=1}^{y=6} \sqrt{1 + \left(\frac{df(y)}{dy}\right)^2}\, dy$$

$$\frac{df(y)}{dy} = \frac{d}{dy}\left((y-1)^{\frac{3}{2}}\right)$$

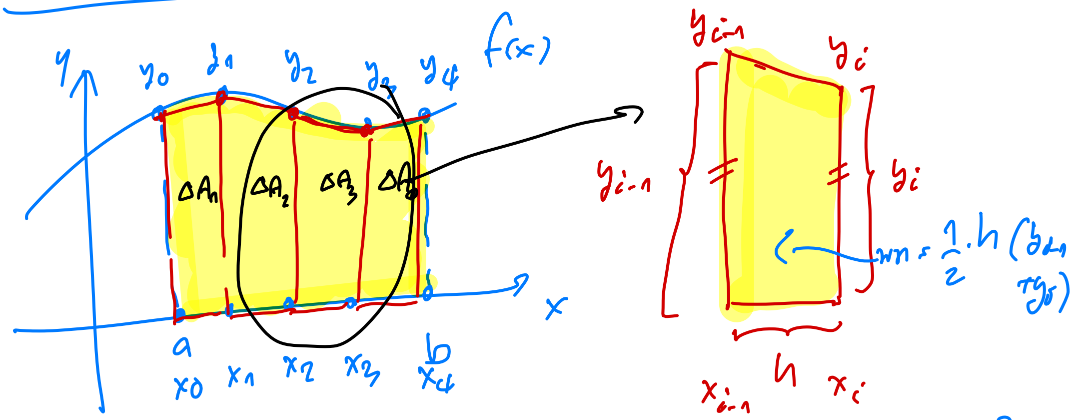
$$= \frac{3}{2}(y-1)^{\frac{1}{2}}$$

$$= \int_{y=1}^{y=6} \sqrt{1+\left[\frac{3}{2}(y-1)^{\frac{1}{2}}\right]^2}\, dy = \int_{y=1}^{y=6} \sqrt{1+\frac{9}{4}(y-1)}\, dy$$

$$= \frac{1}{2}\int_{y=1}^{y=6} \sqrt{4+9y-1}\, dy = \frac{1}{2}\int_{y=1}^{y=6} \sqrt{\underbrace{9y+3}_{u}}\, dy$$

$$= \cancel{2}\,\frac{1}{\cancel{2}}\,\frac{(9y+3)^{\frac{3}{2}}}{3\cdot 9}\Bigg|_{y=1}^{y=6} = \frac{1}{27}\left[(9\cdot 6+3)^{\frac{3}{2}} - (9\cdot 1+3)^{\frac{3}{2}}\right]$$

⇒ หาอินทิกรัลเชิงตัวเลข: (ประมาณค่าอินทิกรัล)



พท. □ คางหมู: $\frac{1}{2}$ × สูง × ผลบวกด้านคู่ขนาน

$$\rightarrow \boxed{\Delta A_i = \frac{h}{2}(y_{i-1}+y_i)}$$

$$A \approx \sum_{i=1}^{N} \Delta A_i = \underbrace{\frac{h}{2}(y_0+y_1)}_{\Delta A_1} + \underbrace{\frac{h}{2}(y_1+y_2)}_{\Delta A_2} + \cdots + \underbrace{\frac{h}{2}(y_{N-2}+y_{N-1})}_{\Delta A_{N-1}} + \underbrace{\frac{h}{2}(y_{N-1}+y_N)}_{\Delta A_N}$$

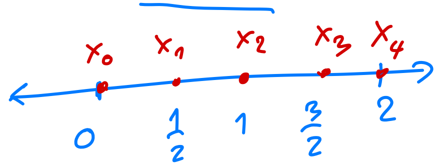
$$\Rightarrow \boxed{A \approx \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \dots + 2y_{N-2} + 2y_{N-1} + y_N\right]}$$

Ex: จงใช้วิธีการประมาณค่า ◻ คางหมู ประมาณ $\int_{x=0}^{x=2} f(x)\,dx$

โดยใช้ข้อมูลต่อไปนี้ : โดยที่ $h = \frac{1}{2}$

i	0	1	2	3	4
x_i	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x_i)$ (y_i)	0	$\frac{5}{16}$	5	$\frac{40}{16}$	20
j	0		1		2

(ถ้าเปลี่ยนความกว้าง)

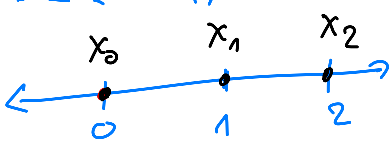


$$\int_{x=0}^{x=2} f(x)\,dx \approx \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$= \frac{\left(\frac{1}{2}\right)}{2}\left[0 + 2\cdot\frac{5}{16} + 2\cdot 5 + 2\cdot\frac{40}{16} + 20\right]$$

$$= \dots$$

ถ้าเลือก $h = 1$:



$h = 1$

$$\int_{x=0}^{x=2} f(x)\,dx \approx \frac{h}{2}\left[y_0 + 2y_1 + y_2\right]$$

$$= \frac{1}{2}\left[0 + 2\cdot 5 + 20\right] = \frac{30}{2} = 15$$