=> การบ้าน: แบบฝึกหัด 3.5

9.) $\int \cos(4\theta)\cos(-3\theta)\,d\theta$

10.) $\int (\sin x + \cos x)^2\,dx$

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9. จาก $\int \cos(4\theta)\cos(-3\theta)\,d\theta$

สูตร: $\cos(mx)\cos(nx) = \frac{1}{2}\left[\cos(m-n)x + \cos(m+n)x\right]$

จะได้ $= \int \frac{1}{2}\left[\cos(4-(-3))\theta + \cos(4+(-3))\theta\right]$

$= \frac{1}{2}\int \cos(7\theta) + \cos(\theta)\,d\theta$

เปลี่ยนตัวแปร.

$= \frac{1}{2}\frac{\sin(7\theta)}{7} + \frac{\sin\theta}{1} + C$ ■

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10.) $\int (\sin x + \cos x)^2\,dx$

กระจาย $= \int \sin^2 x + 2\sin x\cos x + \cos^2 x\,dx$

จัดรูป. $= \int \underbrace{\sin^2 x + \cos^2 x}_{\to 1} + \sin(2x)\,dx$

$$= -\frac{\cos(2x)}{2} + C$$ ■

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\Rightarrow \cos(2x) = 1 - 2\sin^2 x$

$$= -\frac{(1-2\sin^2 x)}{2} + C = \sin^2 x \underbrace{- \frac{1}{2} + C}_{C}$$ ■

ตัวอ. $\Rightarrow \int 2\sin x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$$= \int 2\sin x \, \cancel{\cos x} \, \frac{du}{\cancel{\cos x}}$$

แทนu กลับ.

$$= 2\int u \, du = \frac{\cancel{2} u^2}{\cancel{2}} + C = \sin^2 x + C$$ ■

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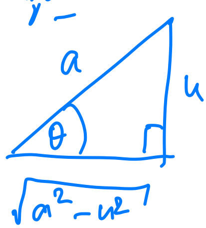
$\Rightarrow$ การอินทิเกรตโดยการแทนค่าด้วยฟังก์ชันตรีโกณมิติ.

รูปแบบ

ความสัมพันธ์ในรูป $\theta$.

- $(a^2 - u^2)^{\frac{m}{2}}$



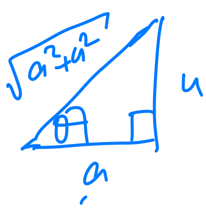

$\frac{u}{a} = \sin\theta$

$\Rightarrow \boxed{u = a\sin\theta}$

$\frac{\sqrt{a^2-u^2}}{a} = \cos\theta$

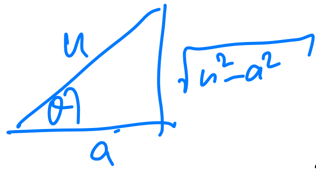
$\Rightarrow \boxed{\sqrt{a^2-u^2} = a\cos\theta}$

- $(a^2 + u^2)^{\frac{m}{2}}$




$\boxed{\begin{array}{l} \bullet\ u = a\tan\theta \\ \bullet\ \sqrt{a^2+u^2} = a\sec\theta \end{array}}$

- $(u^2-a^2)^{\frac{m}{2}}$




$$\boxed{\begin{array}{l} \bullet\ u = a\sec\theta \\ \bullet\ \sqrt{u^2-a^2} = a\tan\theta \end{array}}$$

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Ex: จงหา $\displaystyle\int \frac{1}{(\underbrace{4x^2}_{u}-1)^{3/2}}\,dx$ ให้ $u = 2x$

$du = 2\,dx$

$dx = \frac{du}{2}$

① จัดรูป u

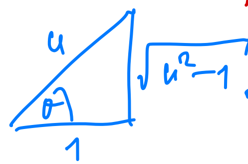
จัดรูป u

$$= \int \frac{1}{(u^2-1)^{3/2}}\,\frac{du}{2}$$

ใช้การแทนค่า เปลี่ยนเป็น θ

$du = \sec\theta\tan\theta\,d\theta$

- $u = 1\sec\theta$




- $\sqrt{u^2-1} = 1\tan\theta$

② ใช้ความสัมพันธ์ตรีโกณ.

จัดรูป θ

$$= \frac{1}{2}\int \frac{1}{(\tan\theta)^{\cancel{3}\,2}}\sec\theta\,\cancel{\tan\theta}\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta = \frac{1}{2}\int \frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta$$

③ อินทิเกรตรูป θ

$$= \frac{1}{2}\int \csc\theta\cot\theta\,d\theta = -\frac{1}{2}\csc\theta + C$$

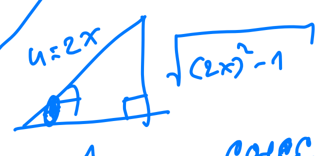
④ เปลี่ยนกลับเป็น x

เปลี่ยน $\theta \to x$

ใช้ความสัมพันธ์ △

$$= -\frac{1}{2}\left(\frac{2x}{\sqrt{(2x)^2-1}}\right) + C \quad \#$$

ความสัมพันธ์ △




$\csc\theta = \frac{2x}{\sqrt{(2x)^2-1}}$

Ex: หาค่า $\displaystyle\int e^x\sqrt{4-\underbrace{e^{2x}}_{=(e^x)^2}}\,dx$ ; $(e^x)^2$ where $e^x = u$

$u = e^x$

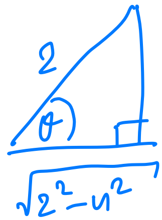
$du = e^x\,dx$

$dx = \dfrac{du}{e^x}$

① เลือก u.

$$= \int \cancel{e^x}\sqrt{2^2-u^2}\,\frac{du}{\cancel{e^x}}$$

② เลือก θ ให้ตรีโกณ.




$\Rightarrow du = 2\cos\theta\,d\theta$

- $u = 2\sin\theta$
- $\sqrt{2^2-u^2} = 2\cos\theta$

$$= \int 2\cos\theta \cdot 2\cos\theta\,d\theta$$

$$= 4\int \cos^2\theta\,d\theta = \cancel{4}^{\,2}\int \frac{1-\cos(2\theta)}{\cancel{2}}\,d\theta$$

③ อินทิเกรตรูป θ

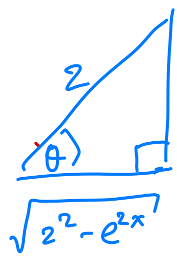
$$= 2\left[\theta - \frac{\sin(2\theta)}{2}\right] + C$$

จัดรูป

$$= 2\left[\theta - \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right] + C$$

④ เปลี่ยน θ → x

$$= 2\left[\arcsin\left(\frac{e^x}{2}\right) - \left(\frac{e^x}{2}\right)\cdot\left(\frac{\sqrt{4-e^{2x}}}{2}\right)\right] + C$$




- $\sin\theta = \dfrac{e^x}{2}$
- $\cos\theta = \dfrac{\sqrt{2^2-e^{2x}}}{2}$
- $\theta = \arcsin\left(\dfrac{e^x}{2}\right)$

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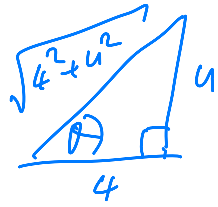
Ex: หาค่า $\displaystyle\int \frac{1}{x\sqrt{\underbrace{16+9x^2}_{4^2+(3x)^2}}}\,dx$ ; $3x \leftarrow u$

① จัดรูป u.   ให้ $u = 3x \Rightarrow du = 3dx \Rightarrow dx = \frac{du}{3}$

$$= \int \frac{1}{x\sqrt{4^2+u^2}}\,\frac{du}{3} \qquad \Bigg|\; u = 3x \Rightarrow x = \frac{u}{3}$$

$$= \int \frac{\cancel{3}}{u}\cdot\frac{1}{\sqrt{4^2+u^2}}\,\frac{du}{\cancel{3}}$$




$\Rightarrow du = 4\sec^2\theta\, d\theta$

$\Rightarrow u = 4\tan\theta$

$\Rightarrow \sqrt{4^2+u^2} = 4\sec\theta$

② จัดรูป θ

$$= \int \frac{1}{(4\tan\theta)(4\sec\theta)}\,du$$

$$= \int \frac{1}{(4\tan\theta)(\cancel{4\sec\theta})}\cdot\cancel{4}\sec^{\cancel{2}}\theta\, d\theta.$$

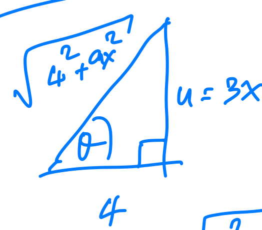
จัดรูป $= \frac{1}{4}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}\,d\theta = \frac{1}{4}\int \operatorname{cosec}\theta\, d\theta$

③ อินทิเกรตรูป θ.

$$= -\frac{1}{4}\ln\left|\operatorname{cosec}\theta + \cot\theta\right| + C$$

④ เปลี่ยน $\theta \to x$

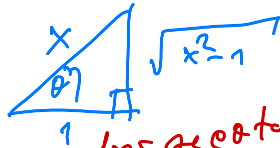
$$= -\frac{1}{4}\ln\left|\frac{\sqrt{4^2+9x^2}}{3x} + \frac{4}{3x}\right| + C$$




- $\operatorname{cosec}\theta = \frac{\sqrt{4^2+9x^2}}{3x}$
- $\cot\theta = \frac{4}{3x}$

แบบฝึกหัด 3.6 :

Ex: 5. หาค่า. $\displaystyle\int \frac{x^2}{(x^2-1)^{5/2}}\,dx$




$\Rightarrow dx = \sec\theta\tan\theta\,d\theta$

- $x = 1\sec\theta$
- $\sqrt{x^2-1} = 1\tan\theta$

② จัดรูป θ

$$= \int \frac{\sec^2\theta}{(\tan\theta)^{\cancel{5}\,4}}\cdot \sec\theta\,\cancel{\tan\theta}\,d\theta$$

$$= \int \frac{\sec^3\theta}{\tan^4\theta}\,d\theta \overset{\text{จัดรูป}}{=} \int \frac{1}{\cancel{\cos^3\theta}}\cdot\frac{\cos^{\cancel{4}}\theta}{\sin^4\theta}\,d\theta$$

③ อินทิเกรต รูป θ.

$u = \sin\theta$

$du = \cos\theta\,d\theta$

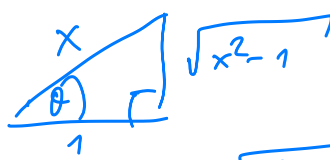
$d\theta = \dfrac{du}{\cos\theta}$

$$= \int \frac{1}{u^4}\cdot\cancel{\cos\theta}\,\frac{du}{\cancel{\cos\theta}}$$

$$= -\frac{1}{3u^3} + C \overset{\text{แทนค่ากลับ}}{=} -\frac{1}{3\sin^3\theta} + C$$

④ เปลี่ยน $\theta \rightarrow x$

$$= -\frac{1}{3}\cdot\left(\frac{x}{\sqrt{x^2-1}}\right)^3 + C \quad \blacksquare$$




- $\sin\theta = \dfrac{\sqrt{x^2-1}}{x}$

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6.) $\displaystyle\int \frac{1}{x\sqrt{2+(\underbrace{\ln x}_{u})^2}}\,dx$

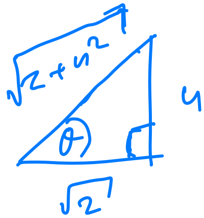
$u = \ln x$

$du = \dfrac{1}{x}\,dx \Rightarrow dx = x\,du$

① เปลี่ยนรูป u

$$= \int \frac{1}{\cancel{x}\sqrt{2+u^2}} \cdot \cancel{x}\, du$$

$\Rightarrow du = \sqrt{2}\sec^2\theta\, d\theta$




- $u = \sqrt{2}\tan\theta$
- $\sqrt{2+u^2} = \sqrt{2}\sec\theta$

② เปลี่ยนรูป θ.

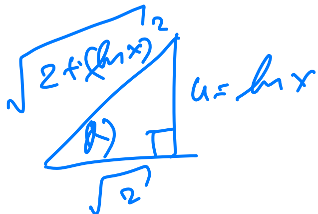
$$= \int \frac{1}{\cancel{\sqrt{2}}\,\cancel{\sec\theta}} \cdot \cancel{\sqrt{2}}\sec^{\cancel{2}}\theta\, d\theta$$

③ อินทิเกรตรูป θ.

$$= \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

④ เปลี่ยน θ → x

$$= \ln\left| \frac{\sqrt{2+(\ln x)^2}}{\sqrt{2}} + \frac{\ln x}{\sqrt{2}} \right| + C \quad \#$$




- $\sec\theta = \dfrac{\sqrt{2+(\ln x)^2}}{\sqrt{2}}$
- $\tan\theta = \dfrac{\ln x}{\sqrt{2}}$

**การบ้าน:** แบบฝึกหัด 3.6.

6.) $\displaystyle\int \frac{1}{(x-1)\sqrt{x^2-2x}}\, dx$

7.) $\displaystyle\int \frac{e^t}{(4-e^{2t})^{3/2}}\, dt.$