การบ้าน: แบบฝึกหัด 3.4

6.) $\int \sin x \ln(\cos x)\, dx$

8.) $\int x^3 e^{x^2} dx$

10.) $\int e^x \arcsin(e^x)\, dx$

11.) $\int \dfrac{xe^x}{(x+1)^2}\, dx$

การบ้าน: แบบฝึกหัด 3.5.7

2.) $\int \sin^5(2x) \cos(2x)\, dx$

5.) $\int \sin^4 x \cos^3 x\, dx$

6) $\int \sin^4(2\theta)\, d\theta$.

การบ้าน: by parts;

6.) $\int \underbrace{\sin x\, dx}_{=dv}\; \underbrace{\ln(\cos x)}_{u}$

$u = \ln(\cos x)$

$\int dv = \int \sin x\, dx$

$v = -\cos x$

$$= (\ln \cos(x))(-\cos x) - \int -\cos(x)\, \underbrace{d \ln(\cos(x))}_{= \frac{1}{\cos x}(-\sin x) dx}$$

$$= -\ln(\cos x) \cos x - \int \cancel{\cos x} \frac{1}{\cancel{\cos x}} \sin x\, dx$$

$$= -\ln(\cos x) \cos x + \cos x + C$$

9.) $\int x^3 e^{x^2}\,dx$ จัดรูป $= \int \underbrace{x^2}_{u} \cdot \underbrace{x e^{x^2} dx}_{dv}$

| $u = x^2$ ของ by parts
| $\int dv = \int x e^{x^2} dx$
| $v = \dfrac{e^{x^2}}{2}$

by parts $= (x^2)\left(\dfrac{e^{x^2}}{2}\right) - \int \dfrac{e^{x^2}}{2}\, d(x^2)$ $(x^2) = u$, $(x^2) = u$.

เปลี่ยนตัวแปร $(u = x^2)$

$= \dfrac{x^2 e^{x^2}}{2} - \int \dfrac{e^u}{2}\,du$

แทน u กลับ.

$= \dfrac{x^2 e^{x^2}}{2} - \dfrac{e^u}{2} + C = \dfrac{x^2 e^{x^2}}{2} - e^{x^2} + \underline{\underline{C}}$

10.) $\int \underbrace{e^x dx}_{dv}\, \underbrace{\arcsin(e^x)}_{u}$

| $u = \arcsin(e^x)$
| $\int dv = \int e^x dx$
| $v = e^x$

by parts:

$= (\arcsin(e^x)) \cdot e^x - \int e^x \, \underbrace{d \arcsin e^x}_{= \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x dx}$

$\left[(e^x)^2 = e^{2x}\right]$

$= e^x \arcsin(e^x) - \int \dfrac{(e^x)^2}{\sqrt{1-(e^x)^2}}\,dx$

เปลี่ยนตัวแปร
| $u = 1 - e^{2x}$
| $du = -2e^{2x} dx$
| $dx = -\dfrac{1}{2\underbrace{e^{2x}}_{=(1-u)}} du$

$= e^x \arcsin(e^x) - \int \dfrac{\cancel{(1-u)}}{\sqrt{u}} \cdot \dfrac{-1}{2\cancel{(1-u)}}\,du$ นั่นคือ $\left[e^{2x} = 1 - u\right]$

$= e^x \arcsin(e^x) + \int \dfrac{1}{\sqrt{u}}\,du$

$$= e^x \arcsin(e^x) + \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

(แทน u กลับ)

$$= e^x \arcsin(e^x) + 2(1-e^{2x})^{\frac{1}{2}} + C \quad \blacksquare$$

11.) $\displaystyle\int \frac{\overbrace{xe^x}^{=u}}{(x+1)^2}\,dx$ $\quad \frac{dx}{(x+1)^2} = dv$

$u = xe^x$

$\displaystyle\int dv = \int \frac{1}{(x+1)^2}\,dx$

$v = -(x+1)^{-1}$

by parts:

$$= (xe^x)\cdot(-(x+1)^{-1}) - \int \frac{-1}{(x+1)}\,\underbrace{d(xe^x)}$$

$= (xe^x + e^x)\,dx$

$= e^x(x+1)\,dx$

$$= \frac{-xe^x}{(x+1)} + \int \frac{1}{\cancel{(x+1)}} \cdot e^x \cancel{(x+1)}\,dx$$

$$= \frac{-xe^x}{(x+1)} + e^x + \underline{C} \quad \blacksquare$$

แบบฝึกหัด 3.5

2.) $\displaystyle\int \sin^5(\underline{2x})\cos(2x)\,dx$

3 แบบ: $\displaystyle\int \sin^m x \cos^n x\,dx$

① m เลขคี่

- $u = \cos x$
- แทน $\sin^2 x = 1 - \cos^2 x$

ถ้า $u = \cos(2x) \Rightarrow du = -2\sin(2x)\,dx \Rightarrow dx = \dfrac{du}{-2\sin(2x)}$

จะได้

$$= \int \underbrace{\sin^4(2x)}_{=(\sin^2(2x))^2} \cdot \cancel{\sin(2x)} \cdot \cos(2x) \cdot \frac{du}{-2\cancel{\sin(2x)}}$$

แทน $\left[\sin^2(2x) = 1 - \cos^2(2x)\right]$

$u = \cos(2x)$

$$= \frac{1}{(-2)} \int \left(1 - \underbrace{\cos^2(2x)}_{u^2}\right)^2 \cdot \underbrace{\cos(2x)}_{u} \, du$$

$$= \frac{-1}{2} \int (1-u^2)^2 \cdot u \, du = \frac{-1}{2} \int (1 - 2u^2 + u^4) \cdot u \, du$$

$$= \frac{-1}{2} \int u - 2u^3 + u^5 \, du = -\frac{1}{2}\left[\frac{u^2}{2} - \frac{2u^4}{4} + \frac{u^6}{6}\right] + C$$

แทน u กลับ

$$= -\frac{1}{2}\left[\frac{\cos^2(2x)}{2} - 2\frac{\cos^4(2x)}{4} + \frac{\cos^6(2x)}{6}\right] + C$$

5.1) $\int \sin^4 x \cos^3 x \, dx$ → แบบที่ 2 : n เลขคี่

- $u = \sin(x)$
- ใช้ $\cos^2 x = 1 - \sin^2 x$

$\left[\rightarrow u = \sin x \Rightarrow du = \cos x \, dx \Rightarrow dx = \frac{du}{\cos x}\right]$

จะได้

$$= \int \sin^4 x \cos^2 x \cdot \cancel{\cos x} \, \frac{du}{\cancel{\cos x}}$$

$(\cos^2 x = 1 - \sin^2 x)$

$$= \int \sin^4 x (1 - \sin^2 x) \, du$$

(จาก $u = \sin x$)

$$= \int u^4 (1-u^2) \, du = \int u^4 - u^6 \, du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C \quad \text{แทน } u \text{ กลับ.} \quad = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$$

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6) $\int \sin^4(2\theta)\, d\theta$ $\Rightarrow$ แนวคิด: m & n เป็นเลขคู่ $(n = 0)$

เตรียม. $\bullet\ \sin^2 x = \frac{1 - \cos(2x)}{2}$

$\bullet\ \cos^2 x = \frac{1 + \cos(2x)}{2}$

จัดรูป

$$= \int \left(\sin^2(2\theta)\right)^2 d\theta$$

$$= \int \left(\frac{1 - \cos(4\theta)}{2}\right)^2 d\theta$$

$$= \frac{1}{4} \int 1 - 2\cos(4\theta) + \underbrace{\cos^2(4\theta)}_{= \frac{1 + \cos(8\theta)}{2}}\, d\theta$$

$$= \frac{1}{4} \int 1 - 2\cos(4\theta) + \frac{1}{2}\left[1 + \cos(8\theta)\right] d\theta$$

$u = 4\theta,\ d\theta = \frac{du}{4}$

$\int \cos \underbrace{4\theta}_{u}\, d\theta$

$= \int \cos(u) \frac{du}{4}$

$= \frac{\sin(u)}{4} = \frac{\sin(4\theta)}{4}$

$$= \frac{1}{4} \int \frac{3}{2} - 2\cos(4\theta) + \frac{1}{2}\cos(8\theta)\, d\theta$$

$$= \frac{1}{4}\left[\frac{3\theta}{2} - 2\,\frac{\sin(4\theta)}{4} + \frac{1}{2}\,\frac{\sin(8\theta)}{8}\right] + C$$

⇒ อินทิกรัลในรูป.

$$\int \tan^m x \sec^n x \, dx$$

โดยใช้: • $\tan^2 x = \sec^2 x - 1$

• $\sec^2 x = \tan^2 x + 1$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}}^{\,1} = \frac{1}{\cos^2 x}$$

$$\Rightarrow \tan^2 x + 1 = \sec^2 x$$

และความรู้ที่ว่า:

• $d \tan x = \sec^2 x \, dx$

• $d \sec x = \sec x \tan x \, dx$

Ex: จงหา. $\int \sec^3 x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$dx = \dfrac{du}{\sec x \tan x}$

แทนค่า

$$= \int \sec^{\cancel{3}\,2} x \, \cancel{\tan x} \, \frac{du}{\cancel{\sec x} \, \cancel{\tan x}}$$

$$= \int \sec^2 x \, du \overset{(u = \sec x)}{=} \int u^2 \, du = \frac{u^3}{3} + C$$

$$\overset{(\text{แทนกลับ})}{=} \frac{\sec^3 x}{3} + C$$

อินทิกรัลในรูป:

1.) $\int \sin(mx) \cos(nx) \, dx$

2.) $\int \sin(mx) \sin(nx) \, dx$

3.) $\int \cos(mx) \cos(nx) \, dx$

ใช้สูตร:

$$\sin(mx)\cos(nx) = \frac{1}{2}\left[\sin((m-n)x) + \sin((m+n)x)\right]$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\left[\cos((m-n)x) - \cos((m+n)x)\right]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left[\cos((m-n)x) + \cos((m+n)x)\right]$$

$\Rightarrow$ ที่มา: $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$ — ①

$$\sin(a-b) = \sin(a)\underbrace{\cos(-b)}_{=\cos(b)} + \cos(a)\underbrace{\sin(-b)}_{-\sin(b)} \quad \text{— ②}$$

ความรู้ที่ใช้เพิ่ม $\left(\cos(-\theta) = \cos\theta, \ \sin(-\theta) = -\sin\theta\right)$

① + ② $\Rightarrow$ $\sin(a+b) + \sin(a-b) = 2\sin(a)\cos(b)$

$$\Rightarrow \sin(\underbrace{a}_{mx})\cos(\underbrace{b}_{nx}) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right] \checkmark$$

Ex: จงหา $\int \sin(3x)\cos(5x)\, dx$

$$= \int \frac{1}{2}\left[\sin((3-5)x) + \sin((3+5)x)\right] dx$$

$$= \frac{1}{2}\int \sin(-2x) + \sin(8x)\, dx$$

$$= \frac{1}{2}\,\frac{-\cos(-2x)}{-2} + \frac{-\cos(8x)}{8} + C$$

Ex: แบบฝึกหัด 3.5

1.) $\int \cos^3 x \sqrt{\sin x}\, dx$, $u = \sin x$

$du = \cos x\, dx$

$dx = \dfrac{du}{\cos x}$

แทน u $= \int \cos^{\cancel{3}\,2} x \; u^{\frac{1}{2}} \dfrac{du}{\cancel{\cos x}}$

$(\cos^2 x = 1-\sin^2 x)$

$= \int (1-u^2)\cdot u^{\frac{1}{2}}\, du = \int u^{\frac{1}{2}} - u^{\frac{5}{2}}\, du$

$= \dfrac{2}{3}u^{\frac{3}{2}} - \dfrac{2u^{\frac{7}{2}}}{7} + C$ = แทน u กลับ $\dfrac{2}{3}(\sin x)^{\frac{3}{2}} - \dfrac{2\sin^{\frac{7}{2}} x}{7} + C$

3.) $\int \cos^2(\pi x)\, dx$ ใช้ $\cos^2(\pi x) = \dfrac{1+\cos(2\pi x)}{2}$

$= \int \dfrac{1+\cos(2\pi x)}{2}\, dx = \dfrac{x}{2} + \dfrac{\sin(2\pi x)}{2\cdot(2\pi)} + C$

7.) $\int \sin(\theta) \sin 3\theta\, d\theta$: สูตร. $\sin(mx)\cdot\sin(nx)$

$= \dfrac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$

$= \int \dfrac{1}{2}\left[\cos((1-3)\theta) - \cos((1+3)\theta)\right] d\theta$

$= \dfrac{1}{2}\int \cos(-2\theta) - \cos(4\theta)\, d\theta.$

$= \dfrac{1}{2}\left[\dfrac{\sin(-2\theta)}{-2} - \dfrac{\sin(4\theta)}{4}\right] + C$

⇒ **การบ้าน**: แบบฝึกหัด 3.5

9.) $\int \cos(4\theta)\cos(-3\theta)\,d\theta$

10.) $\int (\sin x + \cos x)^2\,dx$