(เปลี่ยนตัวแปร.)

การบ้าน: แบบฝึกหัด 3.3

4.) $\displaystyle\int \frac{y}{\sqrt{2y^2+1}}\,dy$

11.) $\displaystyle\int \frac{1}{\sqrt{3+4x-4x^2}}\,dx$

14.) $\displaystyle\int \frac{\ln^2 x}{x}\,dx$

(อินทิกรัลจำกัดเขต.)

แบบฝึกหัด 3.2

1.) $\displaystyle\int_1^4 \frac{1}{t\sqrt{t}}\,dt$

5.) $\displaystyle\int_{-1}^{1} (1-|x|)\,dx$

การบ้าน:

1.) $\displaystyle\int_1^4 \frac{1}{t\sqrt{t}}\,dt = \int_1^4 t^{-\frac{3}{2}}\,dt$

$$= \left(\frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right)\Bigg|_{t=1}^{t=4} = -2\,t^{-\frac{1}{2}}\Bigg|_{t=1}^{t=4}$$

$$= \left[-2\cdot 4^{-\frac{1}{2}} - \left(-2\cdot 1^{-\frac{1}{2}}\right)\right] = \frac{-2}{2} + 2 = 1 \quad \blacksquare$$

5.) $\displaystyle\int_{-1}^{1} (1-|x|)\,dx \implies 1-|x| = \begin{cases} \overbrace{1-(-x)}^{1+x}, & x<0 \\ 1-x, & x \geq 0 \end{cases}$

$$\implies \int_{-1}^{1} (1-|x|)\,dx = \int_{-1}^{0} 1+x\,dx + \int_0^1 1-x\,dx$$

$$= \left(x + \frac{x^2}{2}\right)\Bigg|_{x=-1}^{x=0} + \left(x - \frac{x^2}{2}\right)\Bigg|_{x=0}^{x=1}$$

$$= \left[0 - \left(-1 + \frac{1}{2}\right)\right] + \left[\left(1 - \frac{1}{2}\right) - 0\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \quad \blacksquare$$

4.) $\displaystyle\int \frac{y}{\sqrt{2y^2+1}}\,dy$

11.) $\displaystyle\int \frac{1}{\sqrt{3+4x-4x^2}}\,dx$

14.) $\displaystyle\int \frac{\ln^2 x}{x}\,dx$

เปลี่ยนตัวแปร.

4.) $\displaystyle\int \frac{y}{\sqrt{\underbrace{2y^2+1}_{=u}}}\,dy$

$u = 2y^2+1$

$\dfrac{du}{dy} = 4y \Rightarrow dy = \dfrac{du}{4y}$

แทนค่า $\displaystyle = \int \frac{\cancel{y}}{\sqrt{u}}\frac{du}{4\cancel{y}} = \frac{1}{4}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{\cancel{4}\,2}\,\frac{u^{-\frac{1}{2}+1}}{\cancel{\left(-\frac{1}{2}+1\right)}} + C$

แทนกลับ $\displaystyle = \frac{1}{2}(2y^2+1)^{-\frac{1}{2}} + C \quad \blacksquare$

11.) $\displaystyle\int \frac{1}{\sqrt{\underbrace{3+4x-4x^2}_{=\sqrt{1-u^2}}}}\,dx$

จัดรูป $\displaystyle = \int \frac{1}{\sqrt{\underbrace{4-(2x-1)^2}_{=4\left(1-\frac{(2x-1)^2}{4}\right)}}}\,dx$

$4-(2x-1)^2$

$= 4-(4x^2-4x+1)$

$= 3+4x-4x^2$ ✓

$\displaystyle = \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\underbrace{\frac{2x-1}{2}}_{=u}\right)^2}}\,dx$

$u = \dfrac{2x-1}{2}$

$\dfrac{du}{dx} = 1 \Rightarrow dx = du$

แทนค่า $\displaystyle = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\arcsin(u) + C = \frac{1}{2}\arcsin\left(\frac{2x-1}{2}\right) + C$

แทนกลับ.

$\blacksquare$

1(ข.) $\displaystyle\int \frac{\ln^2 x}{x}\,dx$

$u = \ln x$

$\dfrac{du}{dx} = \dfrac{1}{x} \rightarrow dx = x\,du$

แทน u

$$= \int \frac{u^2}{\cancel{x}}\,\cancel{x}\,du = \frac{u^3}{3} + C$$

แทน u กลับ.

$$= \frac{(\ln x)^3}{3} + C$$

$\Rightarrow$ Integration by parts.

$\Rightarrow$ ดิฟผลคูณ. $\displaystyle\int 1\,d(u.v) = \int u\,dv + \int v\,du$

$$\Rightarrow \quad uv = \int u\,dv + \int v\,du$$

จัดรูป. $\Rightarrow$

$$\boxed{\int \underbrace{u}_{1}\,\underbrace{dv}_{2} = uv - \int v\,du}$$

Ex: $\displaystyle\int \underbrace{\ln x}_{=u}\,\underbrace{dx}_{=dv}$

$u = \ln x \qquad dv = dx$

$v = x \qquad \Rightarrow v = \int dv = \int dx = x$

by parts:

$\left[\int u\,dv = uv - \int v\,du\right]$

$$\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{v} = \ln x \cdot x - \int x\,\underbrace{d(\ln x)}_{=\frac{1}{x}dx}$$

$$= x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - x + C \qquad \blacksquare$$

[**check:** $\frac{d}{dx}(x\ln x - x + C) = \cancel{x\cdot\frac{1}{x}} + \ln x\cdot 1 - \cancel{1} + 0 = \ln x$ $\blacksquare$]

Ex: $\int \underbrace{x}_{u}\,\underbrace{e^{-x}dx}_{dv}$

$u = x$

$v = -e^{-x}$

$\int dv = \int e^{-x}dx$

$v = -e^{-x}$

[by parts: $\int u\,dv = uv - \int v\,du$]

$$= x(-e^{-x}) + \int +e^{-x}dx = -xe^{-x} - e^{-x} + C \qquad \blacksquare$$

(**check:** $\frac{d}{dx}\left[-xe^{-x} - e^{-x} + C\right] = (-x)e^{-x}(-1) + \cancel{e^{-x}(-1)} - \cancel{(-1)e^{-x}}$

$= xe^{-x}$ $\blacksquare$)

Ex: $\int \underbrace{x^2}_{u}\,\underbrace{e^{x}dx}_{dv}$

$u = x^2$

$v = e^x$

$\int dv = \int e^x dx$

$v = e^x$

[by parts: $\int u\,dv = uv - \int v\,du$]

$$= x^2\cdot e^x - \int e^x\,\underbrace{dx^2}_{2x\,dx} = x^2\cdot e^x - 2\int \underbrace{x}_{u}\,\underbrace{e^x dx}_{dv}$$

$u = x$

$\int dv = \int e^x dx$

$v = e^x$

by parts:

$$= x^2\cdot e^x - 2\left[x\cdot e^x - \int e^x dx\right]$$

$$= x^2e^x - 2\left[xe^x - e^x\right] + \underline{\underline{C}}$$

Ex: $\displaystyle\int e^x \sin 2x \, dx = \int \underbrace{\sin(2x)}_{u} \cdot \underbrace{e^x dx}_{dv}$

by parts : $\int u\,dv = uv - \int v\,du$

$$= \sin(2x) \cdot e^x - \int e^x \, d\sin(2x)$$

| $u = \sin 2x$
| $\int dv = \int e^x dx$
| $v = e^x$

$$= \sin(2x) \cdot e^x - 2\int \underbrace{e^x \overbrace{\cos(2x)}^{=u} dx}_{dv}$$

| $u = \cos(2x)$
| $\int dv = \int e^x dx$
| $v = e^x$

$$= \sin(2x) \cdot e^x - 2\left[\cos(2x) \cdot e^x - \int e^x \, d\cos(2x)\right]$$

$\int e^x \sin 2x \, dx$

$$= \sin(2x) \cdot e^x - 2\cos(2x) \cdot e^x + 2(-2)\int e^x \sin(2x) \, dx$$

ย้ายข้าง
จัดรูป

$$5\int e^x \sin(2x)\,dx = (\sin 2x)e^x - 2\cos(2x)e^x$$

ได้! $\displaystyle\int e^x \sin(2x)\,dx = \frac{1}{5}\left[\sin(2x) \cdot e^x - 2\cos(2x)e^x\right] + \underline{C}$

Ex: $\displaystyle\int \underbrace{x^2 \underbrace{\ln x}_{u} dx}_{dv}$

| $u = \ln x$
| $\int dv = \int x^2 dx$
| $v = \dfrac{x^3}{3}$

by parts: $\int u\,dv = uv - \int v\,du$

$$= \ln x\left(\frac{x^3}{3}\right) - \int \frac{x^3}{3}\,\underbrace{d(\ln x)}_{=\frac{1}{x}dx}$$

$$= (\ln x)\frac{x^3}{3} - \frac{1}{3}\int x^2\,dx = (\ln x)\frac{x^3}{3} - \frac{1}{9}x^3 + C \quad \blacksquare$$

แบบฝึกหัด 3.4)

1.) $\int \underbrace{x}_{u}\,\underbrace{e^{2x}\,dx}_{dv}$

$u = x$

$\int dv = \int e^{2x}\,dx$

$v = \frac{e^{2x}}{2}$

by parts:

$$= x\cdot\left(\frac{e^{2x}}{2}\right) - \int \frac{e^{2x}}{2}\,dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C \quad \blacksquare$$

2.) $\int \underbrace{t}_{u}\,\underbrace{\sin t\,dt}_{dv}$

$u = t$

$\int dv = \int \sin t\,dt$

$v = -\cos t$

by parts

$$= t(-\cos t) - \int -\cos(t)\,dt$$

$$= -t\cos t + \sin(t) + C \quad \blacksquare$$

3.) $\int \underbrace{\arctan x}_{u}\,\underbrace{dx}_{dv}$

$u = \arctan x$

$dv = dx \Rightarrow v = x$

(by parts)

$$= x \arctan x - \int x \, \underbrace{d(\arctan x)}_{= \frac{1}{1+x^2}dx}$$

$$= x \arctan x - \int \frac{x}{\underbrace{1+x^2}_{=u}} dx$$

$u = 1+x^2$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

แทนค่า($u = 1+x^2$)

$$= x \arctan x - \int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$= x \arctan x - \frac{1}{2} \ln|u| + C$$

แทน u กลับ

$$= x \arctan x - \frac{1}{2} \ln|1+x^2| + \underline{\underline{C}}$$

การบ้าน: แบบฝึกหัด 3.4

6.) $\int \sin x \ln(\cos x) \, dx$

8.) $\int x^3 e^{x^2} dx$

9.) $\int (\ln x)^2 dx$

11). $\int \frac{x e^x}{(x+1)^2} dx$

สรุป: by parts. $\int 1 \, d(uv) = \int u \, dv + \int v \, du$

$$uv = \int u \, dv + \int v \, du$$

$$\Rightarrow \boxed{\int \underset{①}{u} \, \underset{②}{dv} = uv - \int v \, du}$$

$\Rightarrow$ ม. ① . u

② $dv \Rightarrow v = \int dv = \ldots$