<u>การบ้าน:</u> แบบฝึกหัด 2.5 (1.5 + 1.6 + 2.2)

<u>การบ้าน:</u> <u>แบบฝึกหัด 2.6</u> (1.3 + 2.11 + 3.3).

แบบฝึกหัด 2.5:

1.5. จงหา $\frac{dy}{dx}$ ของ $y = (x^3+2x)^{37}$

$$\begin{tikzcd} y \arrow[d, "\frac{dy}{du}"] \arrow[dd, bend left, "\frac{dy}{dx}"] \\ u \arrow[d, "\frac{du}{dx}"] \\ x \end{tikzcd}$$

$y = u^{37}$, $u(x) = x^3+2x$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d(u^{37})}{du} \cdot \frac{d(x^3+2x)}{dx}$$

$$= 37u^{36} \cdot (3x^2+2)$$

$(u = x^3+2x)$ $$= 37(x^3+2x)^{36} \cdot (3x^2+2)$$

$$\boxed{\begin{aligned} \frac{dy}{dx} &= 37(x^3+2x)^{36} \frac{d}{dx}(x^3+2x) \\ &= 37(x^3+2x)^{36}(3x^2+2) \end{aligned}}$$

1.6. $y = \left(x^3 - \frac{7}{x}\right)^{-2}$

ให้ $y(u) = u^{-2}$, $u(x) = x^3 - \frac{7}{x}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{d(u^{-2})}{du} \cdot \frac{d(x^3 - 7x^{-1})}{dx}$$

$$= -2u^{-3} \cdot (3x^2 + 7x^{-2})$$

$$(u = x^3 - \tfrac{7}{x}) \quad = -2\left(x^3 - \tfrac{7}{x}\right)^{-3}(3x^2 + 7x^{-2}) \qquad \blacksquare$$

2.2 : จงหา $f'(x)$ ของ $f(x) = \underbrace{(5x+8)^{14}}_{①}\underbrace{(x^3+7x)^{12}}_{②}$

(ดิฟของผลคูณ)

$$\frac{df}{dx} = (5x+8)^{14}\frac{d}{dx}(x^3+7x)^{12} + (x^3+7x)^{12}\frac{d}{dx}(5x+8)^{14}$$

(chain rule)

$$= (5x+8)^{14}\,12(x^3+7x)^{11}\frac{d}{dx}(x^3+7x)$$

$$+ (x^3+7x)^{12}\,14(5x+8)^{13}\frac{d}{dx}(5x+8)$$

$$= (5x+8)^{14} \cdot 12 \cdot (x^3+7x)^{11}(3x^2+7)$$

$$+ (x^3+7x)^{12} \cdot 14 \cdot (5x+8)^{13} \cdot (5.) \qquad \blacksquare$$

การบ้าน: แบบฝึกหัด 2.6 $(1.3 + 2.11 + 3.3)$.

1.3. จงหา $f'(x)$ ของ $f(x) = x - 4\operatorname{cosec} x + 2\cot x$

$$\frac{df(x)}{dx} = \frac{d}{dx}(x - 4\operatorname{cosec} x + 2\cot x)$$

$$= \frac{d}{dx}(x) - 4\frac{d}{dx}(\operatorname{cosec} x) + 2\frac{d}{dx}(\cot x)$$

(දිගු.) $= 1 - 4(-\operatorname{cosec} x \cot x) + 2(-\operatorname{cosec}^2 x)$

$= 1 + 4 \operatorname{cosec} x \cot x - 2\operatorname{cosec}^2 x$ ▨

2.11: සොයන $\frac{dy}{dx}$ කරන. $y = \cos^3(\sin 2x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos^3(\underbrace{\sin 2x})\right) \qquad \left[\begin{array}{l} y(u) = u^3 \\ u(w) = \cos(w) \\ w(x) = \sin(2x) \end{array}\right]$$

$$= \frac{dy}{du}\cdot\frac{du}{dw}\cdot\frac{dw}{dx}$$

$$\begin{tikzcd} y \arrow[d, bend left, "\frac{dy}{du}"] \arrow[ddd, bend left=60, "\frac{dy}{dx}"] \\ u \arrow[d, bend left, "\frac{du}{dw}"] \\ w \arrow[d, bend left, "\frac{dw}{dx}"] \\ x \end{tikzcd}$$

$$= \frac{d(u^3)}{du}\cdot\frac{d(\cos(w))}{dw}\cdot\frac{d\sin(2x)}{dx}$$

$$= 3u^2\cdot(-\sin(w))(\cos(2x))\cdot 2 \quad x$$

$(u = \cos w)$ $= -3\cos^2(w)\sin(w)\cos(2x)\cdot 2$

$(w = \sin 2x)$ $= -6\cos^2(\sin 2x)\sin(\sin 2x)\cos(2x)$ ▨

විකල්පය.

$y = \cos^3(\sin(2x)) = \left(\cos(\sin(2x))\right)^3$

$$\frac{dy}{dx} = 3\left(\cos(\sin 2x)\right)^2 \frac{d}{dx}\left(\cos(\sin(2x))\right)$$

$$= 3(\cos(\sin 2x))^2(-\sin(\sin 2x))\frac{d}{dx}(\sin(2x))$$

$$= -3\cos^2(\sin(2x))\sin(\sin(2x))\cos(2x)\frac{d}{dx}(2x)$$

$$= -6\cos^2(\sin(2x))\sin(\sin(2x))\cos(2x) \quad \blacksquare$$

3.3. หา. $\frac{d^2}{dx^2}y$ ของฟังก์ชัน $y = 9\tan(x^2+1)$

ดัง $$\frac{dy}{dx} = \frac{d}{dx}\left(9\tan(x^2+1)\right)$$

$$= 9\sec^2(x^2+1)\underbrace{\frac{d}{dx}(x^2+1)}_{=2x}$$

$$= 18x\sec^2(x^2+1)$$

ดัง. $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\underbrace{18x}_{①}\ \underbrace{\sec^2(x^2+1)}_{②}\right)$$

$$= (18x)\frac{d}{dx}\sec^2(x^2+1) + \sec^2(x^2+1)\frac{d}{dx}(18x)$$

$$= (18x)\left(2\sec(x^2+1)\right)\frac{d}{dx}\left(\sec(x^2+1)\right) + 18\sec^2(x^2+1)$$

$$= 36x\sec(x^2+1)\sec(x^2+1)\tan(x^2+1)\overbrace{\frac{d}{dx}(x^2+1)}^{=2x}$$
$$+ 18\sec^2(x^2+1)$$

$$= 72x^2\sec(x^2+1)\sec(x^2+1)\tan(x^2+1)$$
$$+ 18\sec^2(x^2+1) \quad \blacksquare$$

รูปแบบอนุพันธ์ของ ฟังก์ชัน exponential. $f(x) = e^x$.

หา $\dfrac{df}{dx}$ เมื่อ $f(x) = e^x$

เรารู้: $$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\Rightarrow \frac{d}{dx}(e^x) = \frac{d}{dx}\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)$$

$$= 0 + 1 + \frac{\cancel{2}x}{\cancel{2}} + \frac{\cancel{3}x^2}{\cancel{3}\cdot 2!} + \frac{\cancel{4}x^3}{\cancel{4}\cdot 3!} + \dots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$$

สรุป : $f(x) = e^x \Rightarrow \dfrac{d}{dx} f(x) = \boxed{\dfrac{d}{dx}(e^x) = e^x}$ ∎

chain rule: $$\boxed{\frac{d}{dx}(e^u) = e^u \frac{du}{dx}}$$

ให้ $f(x) = a^x$ หา $\dfrac{d}{dx} f(x) = \dfrac{d}{dx}(a^x)$

$\ln(a^b) = b\ln(a)$

เพราะ $\underline{a^x \overset{(!)}{=} e^{\ln(a^x)}} = e^{x\ln a}$

$$\Rightarrow \frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{x\ln a}\right) = \underbrace{e^{x\ln a}}_{= a^x} \frac{d(x\ln a)}{dx}$$

$$\boxed{\frac{d}{dx}\left(a^x\right) = (\ln a)\, a^x}$$ ← τύπος.

Ex. 1 Να $y'(x)$ όταν $y(x) = \cos\left(e^x + 1\right)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos\left(e^x + 1\right)\right)$$

$$= -\sin\left(e^x + 1\right) \frac{d}{dx}\left(e^x + 1\right)$$

$$= -\sin\left(e^x + 1\right) \cdot e^x \qquad \blacksquare$$

Ex. 2 Βρες $y'(x)$ όταν $y(x) = e^{-\frac{1}{x^2}} + \frac{1}{e^{x^2}}$

$$\Rightarrow y(x) = e^{-x^{-2}} + e^{-x^2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{-x^{-2}} + e^{-x^2}\right)$$

$$= e^{-x^{-2}} \frac{d}{dx}\left(-x^{-2}\right) + e^{-x^2} \frac{d}{dx}\left(-x^2\right)$$

$$= \left(2x^{-3}\right) e^{-x^{-2}} + (-2x)\, e^{-x^2} \qquad \blacksquare$$

⇒ การหาอนุพันธ์โดยปริยาย. (Implicit f^n)

(explicit f^n): $y(x) = \underline{\quad f(x) \quad}$ ← เขียนด้วย x ทั้งหมด.

Ex: $y(x) = \underbrace{x^2 \sin(\cos(2x))}_{\text{เขียนได้ด้วย } x}$

(Implicit f^n): ไม่สามารถ เขียนในรูป explicit ได้ คือ $y = \overset{f(x)}{\downarrow}$ ---

Ex: • $3(x^2+y^2)^2 = 100xy \overset{(!)}{\Longrightarrow} y = \overset{f(x)}{\cdots}$

• $\cos(2x) = e^{xy} \overset{(!)}{\Longrightarrow} y = \overset{f(x)}{\cdots}$

เทคนิค: ใช้ chain rule โดยมองว่า y เป็น f^n ของ x

แล้วคงรูป $\dfrac{dy}{dx}$ รอไว้.

$\boxed{\text{Ex:}}$ หา $y'(x)$ ของ $y^3+y^2-5y-x^2 = -4$

ดิฟทั้งสอง ⇒ $\dfrac{d}{dx}\left(y^3+y^2-5y-x^2\right) = \dfrac{d}{dx}(-4)$

$$\Rightarrow 3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - (2x) = 0$$

จัดรูป. ⇒ $\dfrac{dy}{dx}\left[3y^2+2y-5\right] = 2x$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{(3y^2+2y-5)}$$

Ex: หา $\frac{dy}{dx}$ ของ $3(x^2+y^2)^2 = 100xy$.

ใช้ imp. diff. $\Rightarrow \frac{d}{dx}\left(3(x^2+y^2)^2\right) = \frac{d}{dx}(100xy)$

(มอง y เป็น fn x)

$$\Rightarrow 6(x^2+y^2)\frac{d}{dx}(x^2+y^2) = 100\left(x\frac{dy}{dx}+y\right)$$

$$\Rightarrow 6(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 100x\frac{dy}{dx}+100y.$$

จัดรูป $\Rightarrow \frac{dy}{dx}\left[12y(x^2+y^2)-100x\right] = 100y - 12x(x^2+y^2)$

$$\Rightarrow \frac{dy}{dx} = \frac{\left[100y - 12x(x^2+y^2)\right]}{\left[12y(x^2+y^2)-100x\right]}$$

(หา $\frac{dy}{dx}$ ที่จุด $\overset{x}{(3},\overset{y}{1)} = (x,y)$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(3,1)} = \frac{\left[100\cdot 1 - 12\cdot 3(3^2+1^2)\right]}{12\cdot 1\cdot(3^2+1^2)-100(3)} = \frac{100-36\cdot 10}{12\cdot 10-300} = \frac{-26\cancel{0}}{-18\cancel{0}} = \frac{13}{9}$$

⇒ อนุพันธ์ของฟังก์ชันลอการิทึม. $y = \ln x$ หา $\frac{dy}{dx}$

$$y = \ln x \xrightarrow{\text{ใส่ } e \text{ (ทั้ง)}} e^{y} = e^{\ln x} = x$$

$\Rightarrow e^{y} = x$ (implicit ฟ.)

diff imp: $\frac{d}{dx}(e^{y}) = \frac{d}{dx}(x)$

$$\Rightarrow e^{y}\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{e^{y}} = \frac{1}{x}$$

ดังนั้น. $\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$ สูตร

chain rule: $\boxed{\frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}}$

$$\Rightarrow f(x) = \log_a x \overset{(!)}{=} \frac{\ln x}{\ln a}$$

$$\Rightarrow \frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{x\ln a}$$

สรุป: $\Rightarrow \boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}} \Rightarrow \boxed{\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}}$

สรุป:

① $\frac{d}{dx}(e^x) = e^x \overset{(\text{chain rule})}{\Longrightarrow} \frac{d}{dx}e^u = e^u\frac{du}{dx}$

② Implicit diff: • มอง y เป็น f^n ของ x

• ดิฟ ทั้งสอง ข้าง

• จัดรูป แก้ $\frac{dy}{dx}$

③ $\frac{d}{dx}(\ln x) = \frac{1}{x} \overset{(\text{chain rule})}{\Longrightarrow} \frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$

$\boxed{4}$ $\frac{d}{dx}a^x = a^x \ln a \Longrightarrow \frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$

$\boxed{5}$ $\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a} \Longrightarrow \frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}$

การบ้าน: แบบฝึกหัด 2.9 (1.6 + 1.8)

1. หา $\frac{dy}{dx}$ ของ

1.6) $y = e^{-2x}\cos(2x)$

1.8) $y = 2^{\sec x}$

1.12) $y = \ln(\ln x)$

1.14) $y = 3^{\log_2 x}$

แบบฝึกหัด 2.8:

1. จงหา $\frac{dy}{dx}$ ของ 1.3) $\sin(x^2y^2) = x$

3. จงหา สมการ ของเส้นสัมผัส. 3.3) $(x^2+y^2)^2 = (x-y)^2$ ที่จุด $(1,-1)$