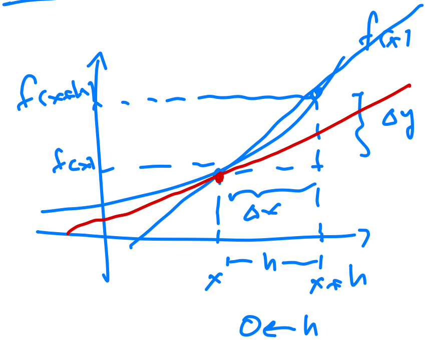
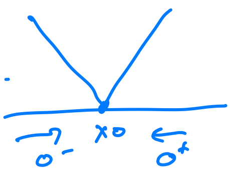
ทบทวน: อนุพันธ์โดยใช้นิยาม.

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ คือ slope ของเส้นสัมผัส.

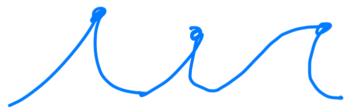


$\Delta y = f(x+h) - f(x)$

$$\boxed{\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$



$f(x) = |x|$ ดิฟไม่ได้



สูตร::

1.) $f(x) = c \Rightarrow f'(x) = 0$

2.) $f(x) = x^n \longrightarrow f'(x) = \frac{d(x^n)}{dx} = n x^{n-1}, n \in \mathbb{R}$

สมบัติ:

1.) $\frac{d}{dx}(k f(x)) = k \frac{d}{dx} f(x)$

2.) $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

3.) $\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$

4.) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$

$(g(x) \neq 0)$

การบ้าน: แบบฝึกหัด 2.3. (9 + 12 + 14).

9.) $y = (x + \frac{1}{x})(x^2 - \frac{1}{x^2})$

$$\Rightarrow \frac{dy}{dx} = \underbrace{(x + x^{-1})}_{f(x)} \cdot \underbrace{(x^2 - x^{-2})}_{g(x)}$$

$$= (x + x^{-1}) \frac{d}{dx}(x^2 - x^{-2}) + (x^2 - x^{-2}) \frac{d}{dx}(x + x^{-1})$$

$$= (x + x^{-1})(2x - (-2)x^{-3}) + (x^2 - x^{-2})(1 - 1 \cdot x^{-2})$$

$$= (x + x^{-1})(2x + 2x^{-3}) + (x^2 - x^{-2})(1 - x^{-2})$$ ■

12.) $y = \frac{5x+1}{2\sqrt{x}}$ $\left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2} \right]$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(5x+1)}{2\sqrt{x}} \right)$$

$u(x)$

$v(x)$

$x^{\frac{1}{2}}$

$$= \frac{(2\sqrt{x}) \frac{d}{dx}(5x+1) - (5x+1) \frac{d}{dx}(2\sqrt{x})}{(2\sqrt{x})^2}$$

$$= \frac{2\sqrt{x}\,(5) - (5x+1)\left(\cancel{2}\left(\frac{1}{\cancel{2}}\right) x^{\left(-\frac{1}{2}\right)}\right)}{4x}$$

$$= \frac{10\sqrt{x} - (5x+1)\, x^{-\frac{1}{2}}}{4x} = \ldots$$ ■

14.) $y = \dfrac{(x+1)(x-2)}{x^2+1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(x+1)(x-2)}{(x^2+1)}\right]$$

$$= \frac{(x^2+1)\frac{d}{dx}\left[(x+1)(x-2)\right] - \left[(x+1)(x-2)\right]\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\left[(x+1)(1) + (x-2)(1)\right] - \left[(x+1)(x-2)\right](2x)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(2x-1) - (x+1)(x-2)(2x)}{(x^2+1)^2} \quad \therefore$$

⇒ อนุพันธ์อันดับสูง (อนุพันธ์อันดับ ≥ 1)

อันดับ 1: ให้ $f(x)$ เป็น ฟังก์ชัน.

$$f'(x) = \frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

อันดับ 2: $(f'(x))' = \dfrac{d}{dx}(f'(x)) = \dfrac{d}{dx}\left(\dfrac{d}{dx}f(x)\right)$

$\Downarrow$

$$f''(x) = \frac{d^2}{dx^2}f(x) \qquad \left[= \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h} \right]$$

อันดับ3: $(f''(x))' = \frac{d}{dx}(f''(x)) = \frac{d}{dx}\left(\frac{d^2}{dx^2}f(x)\right)$

$$\Downarrow$$

$$f'''(x) = \frac{d^3}{dx^3}f(x)$$

อันดับ 4: $f^{(4)}(x) = \frac{d^4}{dx^4}f(x)$

$\vdots$

อันดับ n: $f^{(n)}(x) = \frac{d^n}{dx^n}f(x)$

$\boxed{\text{Ex:}}$ จงหา $f^{(4)}(x)$ เมื่อ $f(x) = \frac{x^4+2}{x}$

แก้ $f'(x) = \frac{d}{dx}\left[\frac{(x^4+2)}{x}\right] = \frac{d}{dx}\left[x^3+2x^{-1}\right]$

$$= 3x^2 - 2x^{-2}$$

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[3x^2-2x^{-2}\right]$$

$$= 6x + 4x^{-3}$$

$$f'''(x) = \frac{d}{dx}\left[f''(x)\right] = \frac{d}{dx}\left[6x+4x^{-3}\right]$$

$$= 6 - 12x^{-4}$$

$$f^{(4)}(x) = \frac{d}{dx}\left[f'''(x)\right] = \frac{d}{dx}\left[6-12x^{-4}\right]$$

$$= 0 + 48x^{-5} \quad ■$$

⇒ กฎลูกโซ่ (Chain rule) ⇒ ใช้กับ composite function

ถ. $f(x), g(x) \Rightarrow f(g(x)) = (f\circ g)(x)$

$\boxed{\text{Ex:}}$ $f(x) = x^{21}$, $g(x) = 2x+1$

$$(f\circ g)(x) = f(g(x)) = f(2x+1) = (2x+1)^{21}$$

$$\boxed{\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)}$$ กฎลูกโซ่

$\boxed{\text{Ex:}}$ $f(x) = x^2$, $g(x) = 2x+1$

$$f(g(x)) = f(2x+1) = (2x+1)^2 = \underline{4x^2+4x+1}$$

① กระจาย ⇒ $\frac{d}{dx}(f(g(x))) = \frac{d}{dx}\left[4x^2+4x+1\right] = 8x+4$ ✓

② chain rule: ⇒ $$\boxed{\frac{d}{dx} f(g(x)) = \underbrace{f'(g(x))}_{①}\, \underbrace{g'(x)}_{②}}$$

โดยที่ $f(x) = x^2 \Rightarrow f'(x) = 2x$

$g(x) = 2x+1 \Rightarrow g'(x) = 2$

จาก chain rule: $$\frac{d}{dx} f(g(x)) = \underbrace{2(2x+1)}_{①} \cdot \underbrace{(2)}_{②}$$

$$= 8x+4$$ ✓

$\boxed{\text{Ex:}}$ $f(x) = x^{21}$, $g(x) = 2x+1$

$f(g(x)) = (2x+1)^{21}$ ⟵ กระจาย ลำบาก

$\Rightarrow$ chain rule: $\boxed{\dfrac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)}$

- $f(x) = x^{21} \Rightarrow f'(x) = 21x^{20}$
- $g(x) = 2x+1 \Rightarrow g'(x) = 2$

จาก chain rule: $\dfrac{d}{dx} f(g(x)) = 21(2x+1)^{20} \cdot 2$ $\square$

$$= 42(2x+1)^{20} \quad \square$$

อีกรูปแบบ. $h(x) = (2x+1)^{21}$

$$\frac{dh(x)}{dx} = \frac{dh(x)}{d(\quad)} \cdot \frac{d(\quad)}{dx} = 21(2x+1)^{20} \cdot \frac{d}{dx}(2x+1)$$

$$= 21(2x+1)^{20} \cdot 2 = 42(2x+1)^{20} \quad \square$$

(หรือ) $h(x) = (2x+1)^{21} = u(x)^{21}$, $u(x) = 2x+1$

$$\Rightarrow \frac{dh(x)}{dx} = \frac{dh(x)}{du(x)} \cdot \frac{du(x)}{dx}$$

$$= 21u^{20} \cdot 2$$

$(u = 2x+1)$ $= 21(2x+1)^{20} \cdot 2 = 42(2x+1)^{20}$.

$\Rightarrow \quad y(u) = u^{100}, \quad u(v) = 2v+1, \quad v(x) = x^5$

สามารถเขียนได้ $\quad y(x) = y(u(v(x)))$

<u>chain rule:</u> $\Rightarrow$

$$\boxed{\frac{d}{dx}y(x) = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}}$$

$\left(\frac{dy}{dx}\right)$

$$\begin{array}{l} y \\ \big\downarrow \tfrac{dy}{du} \\ u \\ \big\downarrow \tfrac{du}{dv} \\ v \\ \big\downarrow \tfrac{dv}{dx} \\ x \end{array} \qquad \frac{dy}{dx}$$

Ex: จงหาอนุพันธ์ของ $\quad y = \sqrt[3]{(x^2+4x+5)}$

$y(u) = \sqrt[3]{u}, \quad u = x^2+4x+5$

chain rule:

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{d}{du}\left(u^{\frac{1}{3}}\right)\cdot\frac{d}{dx}\left(x^2+4x+5\right)$$

$$= \frac{1}{3}u^{-\frac{2}{3}}\cdot(2x+4)$$

$(u = x^2+4x+5) \quad = \frac{1}{3}(x^2+4x+5)^{-\frac{2}{3}}(2x+4)$ ☒

<u>อีกรูปแบบ</u>

$$y = \boxed{(x^2+4x+5)}^{\frac{1}{3}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{3}\boxed{(x^2+4x+5)}^{-\frac{2}{3}}\frac{d}{dx}\boxed{(x^2+4x+5)}$$

$$= \frac{1}{3}(x^2+4x+5)^{-\frac{2}{3}}(2x+4)$$ ☒

การบ้าน: แบบฝึกหัด 2.5 (1.5 + 1.6 + 2.2)

⇒ รูปแบบ: อนุพันธ์ของฟังก์ชันตรีโกณมิติ. ($\sin x$ / $\cos x$)

$$\boxed{\begin{array}{ll} \bullet\ f(x) = \sin(x) \Rightarrow & \frac{d}{dx}(\sin x) = \cos x \\ \bullet\ f(x) = \cos(x) \Rightarrow & \frac{d}{dx}(\cos x) = -\sin x \end{array}}$$

⇒ พิสูจน์. $\frac{d}{dx}\cos(x) = -\sin x$: (โดยใช้ $\frac{d}{dx}\sin x = \cos x$)

$$\Rightarrow \frac{d}{dx}(\cos(x)) = \frac{d}{dx}\left(\sin\left(\frac{\pi}{2} + x\right)\right)$$

(chain rule)

$$= \frac{d}{dx}(\sin(u)) \quad , u = \frac{\pi}{2} + x$$

$$= \frac{d\sin(u)}{du} \cdot \frac{d}{dx}\left(\frac{\pi}{2} + x\right)$$

$$= \cos(u) \cdot 1 \quad \#$$

$$\left(u = \frac{\pi}{2} + x\right) \quad = \cos\left(\frac{\pi}{2} + x\right) = -\sin x \quad \blacksquare$$

$$\left(\cancelto{0}{\cos\left(\frac{\pi}{2}\right)}\cos(x) - \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}\sin(x)\right)$$

$$\Rightarrow \frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$(\text{ดิฟผลหาร.}) = \frac{\cos x \frac{d}{dx}(\sin x) - \sin(x)\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos^2(x) + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

จำ: $\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$

ในทำนองเดียวกัน: $\boxed{\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x}$

$$\Rightarrow \frac{d}{dx}(\sec x) = \frac{d}{dx}\left((\cos x)^{-1}\right)$$

chain rule $$= (-1)\cos(x)^{-2} \cdot (-\sin x)$$

$$= \frac{\sin x}{\cos^2 x} = \left(\frac{1}{\cos x}\right)\cdot\left(\frac{\sin x}{\cos x}\right)$$

$$= \sec x \cdot \tan x.$$

จำ: $\boxed{\frac{d}{dx}\sec x = \sec x \tan x}$

(http: math.science.cmu.ac.th/thawinan)

ในทำนองเดียวกัน: $\boxed{\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x}$

<u>การบ้าน</u>: <u>แบบฝึกหัด. 2.6</u> (1.3 + 2.11 + 3.3).