Newton's Method

(1) $f(x) = 2x - x^2 + 1$

$f'(x) = 2 - 2x$

For $x_0 = 0$, we have

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0 - \frac{1}{2}$$

$$= -\frac{1}{2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= -\frac{1}{2} - \frac{-1/4}{3}$$

$$= -\frac{5}{12}$$

For $x_0 = 2$, we have

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{1}{-2}$$

$$= \frac{5}{2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{5}{2} - \frac{-1/4}{-3}$$

$$= \frac{29}{12}$$

(2) 2.1 Distance between 2 points $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Here, $(x_1, y_1) = (2, -\frac{1}{2})$, $(x_2, y_2) = (x, x^2)$

$$(\text{Distance})^2 = (x-2)^2 + (x^2+\tfrac{1}{2})^2 = f(x)$$

$$f'(x) = 2(x-2) + 2(x^2+\tfrac{1}{2})(2x)$$

To minimize f, we set $f'(x) = 0$

We get

$$2(x-2) + 2(x^2+\tfrac{1}{2})(2x) = 0$$

$$(x-2) + (x^2+\tfrac{1}{2})(2x) = 0$$

$$x - 2 + 2x^3 + x = 0$$

$$2x^3 + 2x - 2 = 0$$

$$x^3 + x - 1 = 0$$

$$x^3 + x = 1$$

$$x(x^2+1) = 1$$

$$x = \frac{1}{x^2+1}$$ as needed.

2.2 Let $g(x) = \frac{1}{x^2+1} - x$

Then, the solution of $g(x) = 0$ is the solution of $x = \frac{1}{x^2+1}$.

$$g'(x) = -1(x^2+1)^{-2}(2x) - 1 = \frac{-2x}{(x^2+1)^2} - 1$$

Let $x_0 = 1$ we get $x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = 1 - \frac{g(1)}{g'(1)} = \frac{2}{3}$

Note that $g(\frac{2}{3}) = \frac{1}{39} \approx 0.02564$

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = \frac{2}{3} - \frac{g(2/3)}{g'(2/3)} = \frac{189}{277} \approx 0.68231$$

Note that $g(\frac{189}{277}) = \frac{883}{31148650} \approx 0.000028348$

which is very close to zero (upto 4 decimal places),

so, this is a good approximation.

$$x = \frac{189}{277} \approx 0.68231$$

3 Let $f(x) = x^3 + 3.6x^2 - 36.4$

we want to solve for $f(x) = 0$ using Newton method.

$$f'(x) = 3x^2 + 7.2x$$

Let $x_0 = 2$, then $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = \frac{167}{66} \approx 2.5303$

Note that $f(x_1) \approx 2.84886$ (still far from 0)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{167}{66} - \frac{f(\frac{167}{66})}{f'(\frac{167}{66})} \approx 2.45418$$

Note that $f(x_2) \approx 0.0644$ (closer to 0 now, but it's good only to 1 decimal place.)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 2.45238$$

Note that $f(2.45238) \approx 0.00003559$ (very close to 0)

so, $x \approx x_3 \approx 2.45238$.