Differential, estimates of change

① $V = \frac{4}{3}\pi r^3 \Rightarrow V'(r) = \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2$

$dV = V' dr \Rightarrow dV = 4\pi r^2 dr$

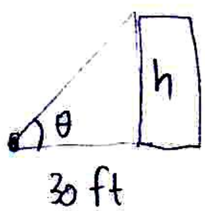
② $V = \pi r^2 h \Rightarrow V'(r) = 2\pi r h$

$dV = V' dr \Rightarrow dV = 2\pi r h\, dr$

when $dr = 0.5$, $h = 30$, $r = 6$,

we get $dV = 2\pi(6)(30)(0.5) = 180\pi$ in^3

③



$\tan\theta = \frac{h}{30} \Rightarrow h = 30\tan\theta$

$dh = 30\sec^2\theta\, d\theta$

$\frac{dh}{h} = \frac{30\sec^2\theta\, d\theta}{30\tan\theta}$

$\Rightarrow \frac{dh}{h} = \frac{1}{\cos\theta\sin\theta} d\theta$ Plug-in $\theta = 75°$, $\frac{dh}{h} = 0.04$

$\Rightarrow 0.04 = \frac{1}{\cos 75° \sin 75°} d\theta$

$\Rightarrow d\theta = 0.04 \cdot \sin 75° \cos 75 = 0.04 \cdot \frac{1}{2}\sin 150° = 0.01$

The angle must be accurate upto 0.01 radian.

i.e. $\theta = 75° \pm 0.01$ radian.

$= 75 \cdot \frac{\pi}{180} \pm 0.01$ radian

$= \frac{5\pi}{12} \pm 0.01$ radian

(4) $W = a + \frac{b}{g} = a + bg^{-1}$

$W'(g) = -bg^{-2} = -\frac{b}{g^2}$

$dW = W'(g)\,dg \Rightarrow dw = -\frac{b}{g^2}\,dg$

On moon, $dW_m = \frac{-b}{(5.2)^2}\,dg$.

On earth, $dW_e = \frac{-b}{(32)^2}\,dg$

$$\frac{dW_m}{dW_e} = \frac{\frac{-b}{(5.2)^2}\,dg}{\frac{-b}{(32)^2}\,dg} = \left(\frac{32}{5.2}\right)^2 = \left(\frac{g_e}{g_m}\right)^2 = 37.8698$$

(5) From #(1), we have $dv = 4\pi r^2 dr$

Here, $dr = 1$ cm

so, $dv = 4\pi(100)^2(1)$

$$\frac{dv}{v} = \frac{4\pi(100)^2}{\frac{4}{3}\pi(100)^3} = \frac{3}{100} \Rightarrow 3\% \text{ change in volume.}$$