(8) D = diameter of the snowball

S = surface area of the snow ball

$S = 4\pi r^2$

$D = 2r$

$\frac{D}{2} = r$

Know $\frac{dS}{dt} = -1 \text{ cm}^2/\text{min}.$

want to find $\frac{dD}{dt}$ when $d = 10$ cm.

Area of surface of sphere : $S = 4\pi r^2 = 4\pi\left(\frac{D}{2}\right)^2$

so, $S = \pi D^2$

Differentiate both sides with respect to t,

we get $\frac{dS}{dt} = \pi \cdot 2D \frac{dD}{dt}$

Plug-in : $-1 = \pi \cdot 2(10) \frac{dD}{dt} \Rightarrow \frac{dD}{dt} = \frac{-1}{20\pi}$

$\frac{dD}{dt} \approx 0.0159$ cm/min

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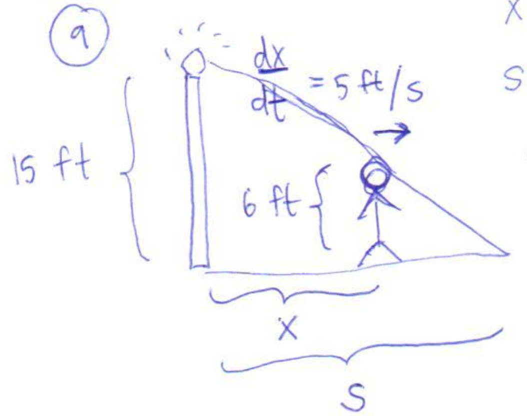
(9)



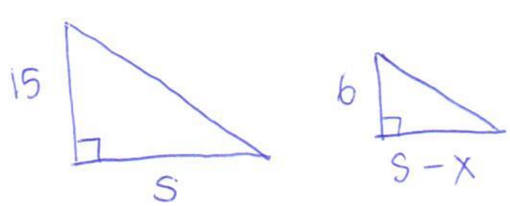
x = distance from the pole to the man

S = distance from the pole to the tip of shadow.

so, in this case we have

$\frac{dx}{dt}$ = speed of the man

$\frac{dS}{dt}$ = speed of the tip of his shadow.



Using similar triangle, we get

$\frac{S}{S-x} = \frac{15}{6} \Rightarrow 6S = 15S - 15x$

$\Rightarrow 15x = 9S$

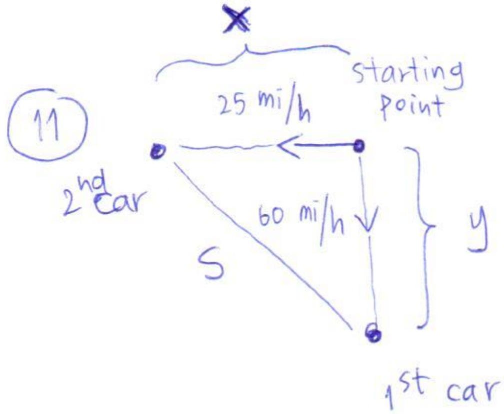
$\Rightarrow x = \frac{9}{15}S = \frac{3}{5}S$

Therefore, after differentiate the equation $x = \frac{3}{5}S$,

we get $\frac{dx}{dt} = \frac{3}{5}\frac{dS}{dt}$. Plug-in : $5 = \frac{3}{5}\frac{dS}{dt} \Rightarrow \frac{dS}{dt} = \frac{25}{3}$ ft/s.

$\cancel{5 = \frac{3}{5} dS}$

11



x = distance from starting point to 2nd car

y = distance from starting point to 1st car

S = distance between two cars.

Know $\frac{dx}{dt} = 25$ mi/h, $\frac{dy}{dt} = 60$ mi/h.

Want to find $\frac{ds}{dt}$ after 2 hours.

Note that after 2 hours, $x = 50$ mi, $y = 120$ mi

and $S = \sqrt{50^2+120^2} = 130$ mi.

From Pythagorean theorem, we have $x^2+y^2 = S^2$

Diff. $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = s\frac{ds}{dt}$$

Plug-in $50(25) + (120)(60) = (130)\frac{ds}{dt}$

so, $\frac{ds}{dt} = \frac{50(25)+(120)(60)}{130} = 65$ mi/h

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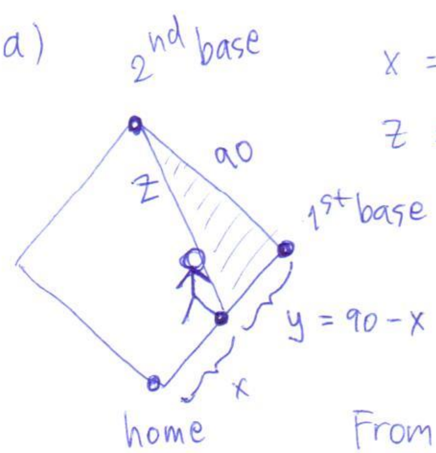
14 (a)



x = distance from ~~the 1st b~~ home to the 1st base.

z = distance from player to the 2nd base

Know $\frac{dx}{dt} = 24$ ft/s.

Want to find $\frac{dz}{dt}$ when $x = 45$ ft

From the shaded region, we have

$$z^2 = 90^2 + y^2 \Rightarrow z^2 = 90^2 + (90-x)^2$$

When $x = 45$, $y = 45$

so, $z = \sqrt{90^2+45^2}$

$z = 45\sqrt{5}$

Diff. $2z\frac{dz}{dt} = 0 + 2(90-x)(-1)\frac{dx}{dt}$

$$2z\frac{dz}{dt} = -2(90-x)\frac{dx}{dt}$$

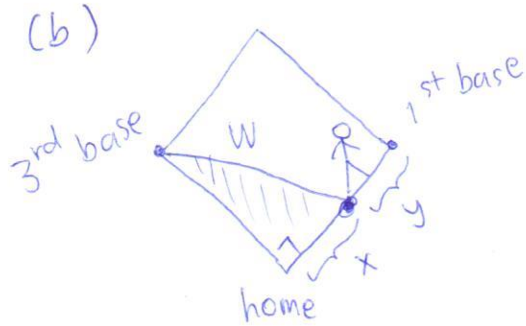
$$z\frac{dz}{dt} = -(90-x)\frac{dx}{dt}$$

Plug-in: $45\sqrt{5}\,\frac{dz}{dt} = -(90-45)(24)$

so, $\frac{dz}{dt} = -\frac{24}{\sqrt{5}}$ ft/s

$\frac{dz}{dt} \approx -10.73$ ft/s

(14) (b)



x = distance from home to the 1^{st} base

w = distance from player to the 3^{rd} base

Know $\frac{dx}{dt} = 24$ ft/s

Want to find $\frac{dw}{dt}$ when $x = 45$ ft.

From the shaded region, we have $x^2 + 90^2 = w^2$

so, when $x = 45$, we get $w = \sqrt{45^2 + 90^2} = 45\sqrt{5}$

From $x^2 + 90^2 = w^2 \Rightarrow$ diff. w.r.t. t to get

$$2x\frac{dx}{dt} + 0 = 2w\frac{dw}{dt} \Rightarrow x\frac{dx}{dt} = w\frac{dw}{dt}$$

Plug-in : $45(24) = 45\sqrt{5}\frac{dw}{dt}$, so $\frac{dw}{dt} = \frac{24}{\sqrt{5}} \approx 10.73$ ft/s

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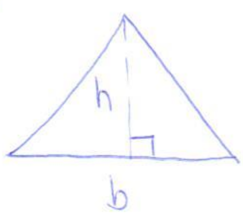
(15)



A = area of Δ

h = altitude (height) of Δ

b = base of Δ

Know $\frac{dh}{dt} = 1$ cm/min, $\frac{dA}{dt} = 2$ cm²/min.

Want to find $\frac{db}{dt}$ when $h = 10$ and $A = 100$.

(When $A = 100$ and $h = 10$, we have that $b = 20$ because $A = \frac{1}{2}bh$.)

Area formula $A = \frac{1}{2}bh$

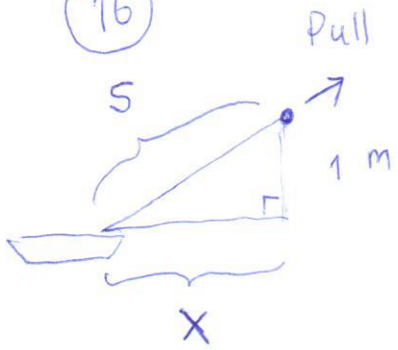
Diff. $\frac{dA}{dt} = \frac{1}{2}b\frac{dh}{dt} + \frac{1}{2}h\frac{db}{dt}$

Plug-in $2 = \frac{1}{2}(20)(1) + \frac{1}{2}(10)\frac{db}{dt}$

$2 = 10 + 5\frac{db}{dt}$

so, $\frac{db}{dt} = \frac{2-10}{5} = -\frac{8}{5} = -1.6$ cm/min.

(16)



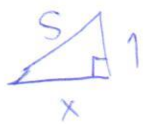
x = distance from the boat to the dock

S = distance from the pulley to the bow of the boat.

In this case, $\frac{dx}{dt}$ = how fast the boat is approaching the dock.

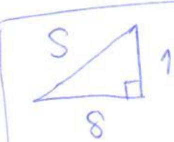
$\frac{ds}{dt}$ = how fast the rope is being pulled in.

Know $\frac{ds}{dt} = -1$ m/s, want to find $\frac{dx}{dt}$ when $x = 8$ m.

From  , we have $S^2 = 1^2 + x^2$.

Diff. $2S\frac{ds}{dt} = 0 + 2x\frac{dx}{dt}$

$S\frac{ds}{dt} = x\frac{dx}{dt}$



when $x = 8$, we get $S = \sqrt{1^2 + 8^2} = \sqrt{65}$

plug-in : $\sqrt{65}(-1) = 8\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{\sqrt{65}}{8} \approx -1.0078$ m/s.

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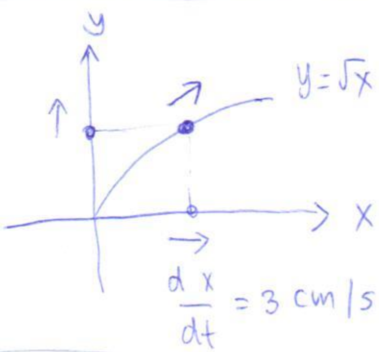
(18)



$y = \sqrt{x}$

Treat x, y as functions of t.

$\frac{dy}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt}$

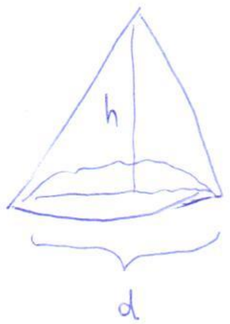
Know $\frac{dx}{dt} = 3$

Want to find $\frac{dy}{dt}$ when $(x, y) = (4, 2)$

plug-in

$\frac{dy}{dt} = \frac{1}{2\sqrt{4}}(3) = \frac{3}{4}$ cm/s

(23)



V = Volume of the cone

h = height of the cone

d = diameter of the cone $= 2r$

In this problem, $h = d$, so $h = 2r \Rightarrow r = \frac{h}{2}$.

Know $\frac{dV}{dt} = 30\ ft^3/min$, want to find $\frac{dh}{dt}$ when $h = 10$.

Volume formula $\quad V = \frac{1}{3}\pi r^2 h$, but $r = \frac{h}{2}$

so, $\quad V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h \Rightarrow V = \frac{1}{12}\pi h^3$

Diff. $\quad \frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \frac{dh}{dt} \Rightarrow \frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

Plug-in $\quad 30 = \frac{1}{4}\pi(10)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{30}{25\pi} \approx 0.382\ ft/min$

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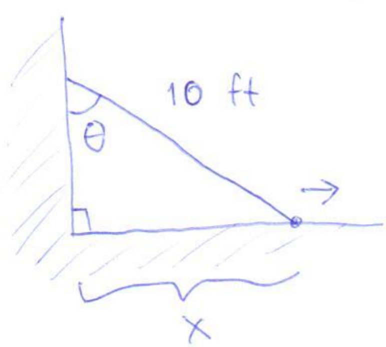
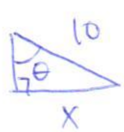
(31)



x = distance between the bottom of the ladder and the wall

θ = angle between the top of the ladder and the wall

Know $\frac{dx}{dt} = 2\ ft/s$, want to find $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{4}$.

From 

, we have that $\sin\theta = \frac{x}{10}$

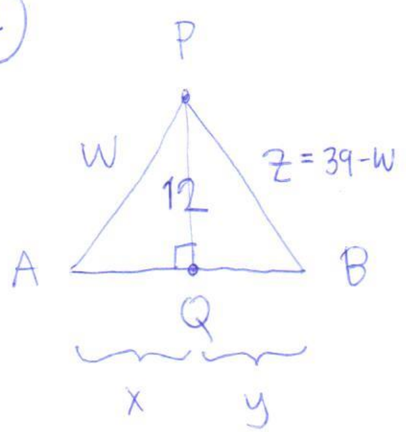
Diff. $\quad \cos\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$

Plug-in $\quad \cos\frac{\pi}{4}\frac{d\theta}{dt} = \frac{1}{10}(2)$

$\frac{1}{\sqrt{2}}\frac{d\theta}{dt} = \frac{1}{5}$

so, $\quad \frac{d\theta}{dt} = \frac{\sqrt{2}}{5} \approx 0.283\ rad/s$

32



x = distance from cart A to Q

y = distance from Q to cart B

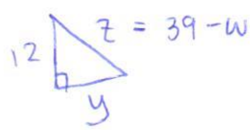
w = distance from A to P, Z = from B to P

(Note that w + z = 39 ft.)

know $\frac{dx}{dt} = 2$ ft/s, want to find $\frac{dy}{dt}$ when $x = 5$ ft

From 

, we have $w^2 = x^2 + 12^2 \Rightarrow w = \sqrt{x^2 + 144}$

From 

, we have $(39 - w)^2 = y^2 + 12^2$

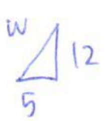
$$\left(39 - \sqrt{x^2+144}\right)^2 = y^2 + 144$$

Differentiate both sides ;

$$2\left(39 - \sqrt{x^2+144}\right) \cdot \frac{-1}{2\sqrt{x^2+144}} \cdot 2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

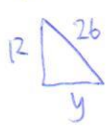
$$\left(39 - \sqrt{x^2+144}\right) \cdot \frac{-x}{\sqrt{x^2+144}} \frac{dx}{dt} = y \frac{dy}{dt}$$

When $x = 5$, $w = \sqrt{12^2 + 5^2} = 13$



so, $z = 39 - 13 = 26$

so, $y = \sqrt{26^2 - 12^2} = 2\sqrt{133}$



Plug-in :

$$\left(39 - \sqrt{5^2+144}\right) \cdot \frac{-5}{\sqrt{5^2+144}} \cdot (2) = 2\sqrt{133} \frac{dy}{dt}$$

$$-20 = 2\sqrt{133} \frac{dy}{dt}$$

so, $\frac{dy}{dt} = -\frac{10}{\sqrt{133}} \approx -0.867$ ft/s.