Rates of change & Tangents to curves

Instantaneous rate of change

(1) 1.1 $P = (20, 650)$, $Q_1 = (10, 220)$ → approximate from graph

So, the slope of PQ_1 is $\frac{650 - 220}{20 - 10} = \frac{430}{10} = 43$

The unit is meter / second.

Similarly, slopes of PQ_2, PQ_3, PQ_4 can be approximated as :

$$m_{PQ_2} \approx \frac{650 - 380}{20 - 14} = \frac{270}{6} = 15 \text{ m/sec}$$

$$m_{PQ_3} \approx \frac{650 - 480}{20 - 17} = \frac{170}{3} \approx 58.67 \text{ m/s}$$

$$m_{PQ_4} \approx \frac{650 - 550}{20 - 18} = \frac{100}{2} = 50 \text{ m/s}$$

1.2 The speed at $t = 20$ sec. is approximately 50 m/s.

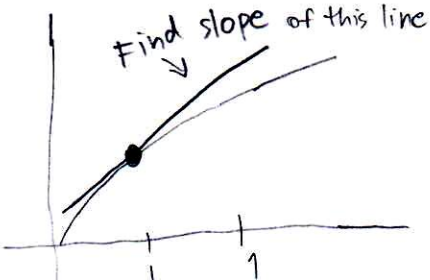
(2) 2.1 Average speed over $[0, 1] = \frac{15 - 0}{1 - 0} = 15$ mi/hr

Avg. speed over $[1, 2.5] = \frac{20 - 15}{2.5 - 1} = \frac{5}{1.5} \approx 3.33$ mi/hr

Avg. speed over $[2.5, 3.5] = \frac{30 - 20}{3.5 - 2.5} = \frac{10}{1} = 10$ mi/hr

2.2 To find instantaneous speed at $t = \frac{1}{2}$, you have to draw a tangent line at that point and find the slope of that line.

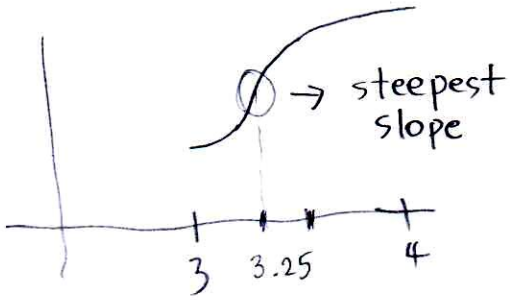
The answer varies, depending on how you draw the line.



Use the same method for $t = 2$, $t = 3$.

2.3 The maximum speed occurs at the point where slope is highest (steepest)

Looking at the graph, slope is highest at around $t = 3.25$



(3) similar to problem (2).

(4) $s(t) = 100 - 4.9t^2$

$v(t) = s'(t) = -9.8t$

At $t = 2$, $v(2) = -9.8(2) = -19.6$ m/sec.

(5) $s(t) = 3t^2$ ft

$v(t) = s'(t) = 6t$, at $t = 10$, $v = 6(10) = 60$ ft/sec.

(6) $A(r) = \pi r^2 \Rightarrow A'(r) = 2\pi r$

At $r = 3$, $A'(3) = 2\pi(3) = 6\pi$

(7) $V(r) = \frac{4}{3}\pi r^3 \Rightarrow V'(r) = 4\pi r^2$

At $r = 2$, $V'(r) = 4\pi(2^2) = 16\pi$