1.1) $\lim_{\Delta x \to 0^-} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{(1+\Delta x)^2 - \frac{1}{1}}{\Delta x}$

$= \lim_{\Delta x \to 0^-} \frac{1 + 2\Delta x + (\Delta x)^2 - 1}{\Delta x}$

$= \lim_{\Delta x \to 0^-} (2 + \Delta x)$

$= 2$

1.2) $\lim_{\Delta x \to 0^+} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{\frac{1}{1+\Delta x} - \frac{1}{1}}{\Delta x}$

$= \lim_{\Delta x \to 0^+} \frac{1 - 1 - \Delta x}{(1+\Delta x)\Delta x}$

$= \lim_{\Delta x \to 0^+} \frac{-1}{1+\Delta x}$

$= -1$

1.3) $f'(1)$ does not exist.

1.4) $f'(4) = \lim_{\Delta x \to 0} \frac{f(4+\Delta x) - f(4)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{4+\Delta x} - \frac{1}{4}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{4 - 4 - \Delta x}{(4+\Delta x)(\Delta x)}$

$= \lim_{\Delta x \to 0} \frac{-1}{4+\Delta x}$

$= -\frac{1}{4}$

2) $T(t) = 65 - 11t + \frac{4}{t}$

$T'(t) = -11 - \frac{4}{t^2}$

$T'(1) = -11 - \frac{4}{1^2} = -15$

3) $y = 2e^{(1-x)}$

$y' = -2e^{1-x}$

$y'|_{x=1} = -2$ At point $(1,2)$, the slope of the curve is -2

Tangent line: $y - 2 = -2(x-1)$

④ $f(a+dx) - f(a) \approx df = f'(a)dx$

Let $a = 1$, $dx = 0.1$, $f(x) = 2^x$, $f'(x) = 2^x \ln 2$.

$f(1+0.1) - f(1) \approx f'(1)(0.1)$

$2^{1.1} - 2^1 \approx (2^1 \ln 2)(0.1)$

$2^{1.1} \approx (2\ln 2)(0.1) + 2 = 2(0.69)(0.1) + 2 = 2.138$

⑤ $V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 \, dr$

$r = 3$, $dr = 0.02$; $dV = 4\pi(3)^2(0.02) = 0.72\pi$

Error $= 0.72\pi$

⑥ 6.1 $f'(x) = 2x - 2\cos(-2x)$

6.2 $f''(x) = 2 - 4\sin(-2x)$

6.3 $f'''(x) = 8\cos(-2x)$

$$\left.\frac{d^3y}{dx^3}\right|_{x=0} = 8\cos[(-2)(0)] = 8$$

⑦ 7.1 $y' = (x + \sqrt{3x})(2e^{2x}) + (e^{2x})(2x + \frac{3}{2\sqrt{3x}})$

7.2 $$y' = \frac{\cos(3x+1)\cdot\frac{3}{3x+1} + 3\ln(3x+1)\sin(3x+1)}{\cos^2(3x+1)}$$

7.3 $y' = \frac{1}{(x^5-2x+3)\ln 2} \cdot (5x^4 - 2) + [\sec(2^x-1)\tan(2^x-1)] \cdot 2^x \ln 2$

7.4 $y' = \frac{-1}{\sqrt{1-(\frac{1}{2x+1})^2}} \cdot \frac{-2}{(2x+1)^2}$

7.5 $y' = 10(\tan(e^x+\pi))^9 \cdot \sec^2(e^x+\pi) \cdot e^x$

⑧ $$y = \frac{(x^2+1)^{\sin x}}{\ln x}$$

$$\ln y = \sin x \ln(x^2+1) - \ln(\ln x)$$

$$\frac{1}{y} \cdot y' = \sin x \cdot \frac{2x}{x^2+1} + \ln(x^2+1)\cos x - \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$y' = \left(\sin x \cdot \frac{2x}{x^2+1} + \ln(x^2+1)\cos x - \frac{1}{x\ln x} \right) \frac{(x^2+1)^{\sin x}}{\ln x}$$

⑨ $$\sqrt{2x+y+1} = y \sin x$$

$$\frac{2+y'}{2\sqrt{2x+y+1}} = y\cos x + y'\sin x$$

$$2+y' = (y\cos x + y'\sin x)(2\sqrt{2x+y+1})$$

$$2+y' = (2y\cos x)\sqrt{2x+y+1} + 2y'(\sin x)\sqrt{2x+y+1}$$

$$(1 - 2(\sin x)\sqrt{2x+y+1})y' = (2y\cos x)\sqrt{2x+y+1} - 2$$

$$y' = \frac{(2y\cos x)\sqrt{2x+y+1} - 2}{1 - (2\sin x)\sqrt{2x+y+1}}$$

⑩ 10.1 $$\lim_{x \to +\infty} \frac{x^2+3x-1}{3-2x^2} = \lim_{x \to +\infty} \frac{1 + \frac{3}{x} - \frac{1}{x^2}}{\frac{3}{x^2} - 2} = -\frac{1}{2}$$

10.2 $$\lim_{x \to +\infty} \frac{1+x^2}{5-3\sin x}$$

$$= +\infty$$

$$-1 \le \sin x \le 1$$

$$-3 \le 3\sin x \le 3$$

$$-3 \le -3\sin x \le 3$$

$$2 \le 5-3\sin x \le 8$$

$$\frac{1}{8} \le \frac{1}{5-3\sin x} \le \frac{1}{2}$$

So, $$\frac{1+x^2}{8} \le \frac{1+x^2}{5-3\sin x} \le \frac{1+x^2}{2}$$

As $x \to \infty$, we have that

$\frac{1+x^2}{8} \to \infty$ and $\frac{1+x^2}{2} \to \infty$.

10.3 $\lim_{x\to 0^+} \left(3 - \frac{1}{x} + \ln x\right) = 3 - (+\infty) + (-\infty) = -\infty$

(11) $\lim_{x\to 0} \frac{\ln(\cos x)}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$\therefore \lim_{x\to 0} \frac{\ln(\cos x)}{x^2} = \lim_{x\to 0} \frac{\frac{-\sin x}{\cos x}}{2x} = \lim_{x\to 0} \frac{-\tan x}{2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \frac{-\sec^2 x}{2}$

$= -\frac{1}{2}$

(12) $\lim_{x\to 0^+} x^2 e^{1/x^2} = \lim_{x\to 0^+} \frac{e^{1/x^2}}{\frac{1}{x^2}}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$\therefore \lim_{x\to 0^+} \frac{e^{1/x^2}}{\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{e^{1/x^2} \cdot (-2)\frac{1}{x^3}}{-2 \cdot \frac{1}{x^3}} = \lim_{x\to 0^+} e^{1/x^2} = +\infty$

(13) $\lim_{x\to 0^+} (1+3x)^{1/x}$ $\left(1^\infty \text{ form}\right)$

Let $y = (1+3x)^{1/x}$

$\ln y = \frac{1}{x} \ln(1+3x)$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{1}{x} \ln(1+3x) = \lim_{x\to 0^+} \frac{\ln(1+3x)}{x}$ $\left(\frac{0}{0} \text{ form}\right)$

$\therefore \lim_{x\to 0^+} \frac{\ln(1+3x)}{x} = \lim_{x\to 0^+} \frac{3}{1+3x} = 3$

$\lim_{x\to 0^+} \ln y = 3 \Rightarrow \ln \lim_{x\to 0^+} y = 3 \Rightarrow \lim_{x\to 0^+} y = e^3$

(14) $f(x) = \ln(2x+1)$ $\qquad f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{2x+1} \cdot 2 = 2(2x+1)^{-1}$ $\qquad f'(0) = 2$

$f''(x) = -2(2x+1)^{-2} = -4(2x+1)^{-2}$ $\qquad f''(0) = -4$

$f'''(x) = 8(2x+1)^{-3} \cdot 2 = 16(2x+1)^{-3}$ $\qquad f'''(0) = 16$

$f^{(4)}(x) = -48(2x+1)^{-4} \cdot 2 = -96(2x+1)^{-4}$ $\qquad f^{(4)}(0) = -96$

(a) $P_4(x) = 0 + 2x - \frac{4}{2!}x^2 + \frac{16}{3!}x^3 - \frac{96}{4!}x^4$

$= 2x - 2x^2 + \frac{8}{3}x^3 - 4x^4$

(b) Note that $f^{(n)}(x) = (-1)^{n-1}(n-1)!\,2^n(2x+1)^{-n}$

and that $f^{(n)}(0) = (-1)^{n-1}(n-1)!\,2^n$

So, $P(x) = \sum_{n=1}^{\infty} (-1)^{n-1}(n-1)!\,2^n$

(c) $P_3(x) = 2x - 2x^2 + \frac{8}{3}x^3$

$\ln(1.2) = f(0.1) \approx P_3(0.1) = 2(0.1) - 2(0.1)^2 + \frac{8}{3}(0.1)^3$

$= 0.2 - 0.02 + \frac{0.008}{3}$

$= 0.2 - 0.02 + 0.0026666$

$= 0.1826666\ldots$

≈ 0.1827

(15) $f(x) = 3x^5 - 5x^3 + 3$

$f'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1)$

$f'(x) = 15x^2(x-1)(x+1)$

set $f'(x) = 0$, we get $15x^2(x-1)(x+1) = 0$

$\Rightarrow x = 0, 1, -1$

These are critical points of f

sign of f'

$$\begin{array}{cccccccc} & \nearrow & & \searrow & & \searrow & & \nearrow \\ & + & 0 & - & 0 & - & 0 & + \\ \hline & & -1 & & 0 & & 1 & \quad x \end{array}$$

local max. local min.

So, f has local max. at $x = -1$, local maximum value $= f(-1) = -3+5+3 = 5$

f has local min. at $x = 1$, local minimum value $= f(1) = 3-5+3 = 1$

(16) $f(x) = 2\sin x - \cos x$

(a) $f(0) = 2\sin 0 - \cos 0 = 0 - 1 = -1 < 0$

$f(\frac{\pi}{2}) = 2\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = 2\cdot 1 - 0 = 2 > 0$

So, there is a solution to $f(x) = 0$ in between 0 and $\frac{\pi}{2}$.

(b) $f'(x) = 2\cos x + \sin x$

Using $x_0 = \frac{\pi}{4}$, we get

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= \frac{\pi}{4} - \frac{2\sin\frac{\pi}{4} - \cos\frac{\pi}{4}}{2\cos\frac{\pi}{4} + \sin\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \frac{\frac{\sqrt{2}}{2}}{\frac{3\sqrt{2}}{2}}$$

$$= \frac{\pi}{4} - \frac{1}{3}$$