30. (a) $\sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$

$$= n \frac{\sum_{i=1}^{n} x_i}{n} - n\bar{x} = n\bar{x} - n\bar{x} = 0$$

(b) $S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} (x_i^2 - 2x_i \bar{x} + \bar{x}^2)}{n}$

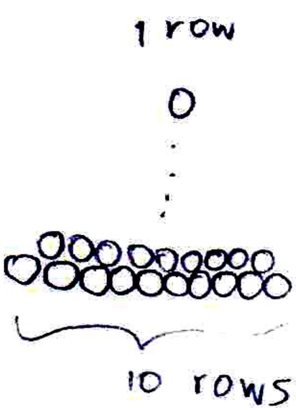
$$= \frac{\sum_{i=1}^{n} x_i^2}{n} - 2\bar{x} \frac{\sum_{i=1}^{n} x_i}{n} + \frac{n\bar{x}^2}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i^2}{n} - 2\bar{x} \cdot \bar{x} + \bar{x}^2$$

$$= \frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2$$

31.

Number of oranges



$$= 1 \cdot 16 + 2 \cdot 16 + \dots + 10 \cdot 16$$

$$= \sum_{i=1}^{10} i \cdot 16$$

$$= 16 \sum_{i=1}^{10} i$$

$$= 16 \frac{(10)(11)}{2}$$

$$= 880$$

32. (a) $\int_{10}^{20} v'(t)\,dt = \int_{10}^{20} (20-t)\,dt$

$= \left[20t - \frac{1}{2}t^2\right]_{10}^{20}$

$= 200 - 150 = 50$ gallons

(b) $\int_{0}^{\tau} v'(t)\,dt = \int_{0}^{\tau} (20-t)\,dt$

$= \left[20t - \frac{1}{2}t^2\right]_{0}^{\tau}$

$= 20\tau - \frac{1}{2}\tau^2 \overset{\text{set}}{=} 200$

$\Rightarrow \tau^2 - 40\tau + 400 = 0$

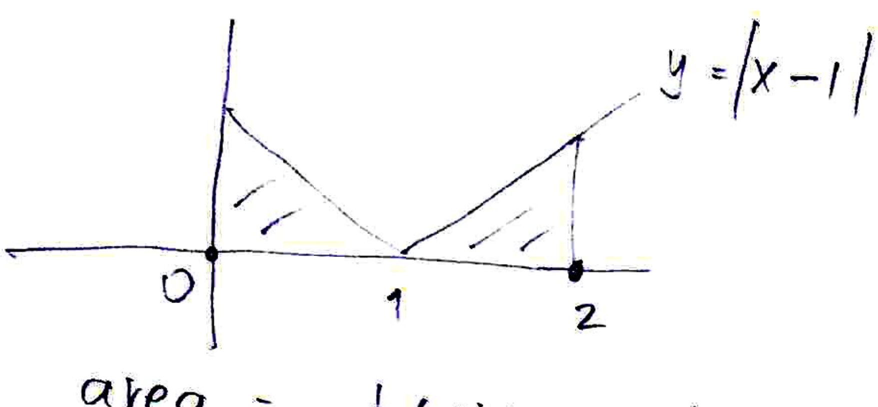
$\Rightarrow (\tau - 20)^2 = 0 \Rightarrow \tau = 20$ hrs

33. $av(f) = \frac{1}{2-(-2)} \int_{-2}^{2} (-x+2)\,dx$

$= \frac{1}{4}\left[-\frac{1}{2}x^2 + 2x\right]_{-2}^{2} = 2$

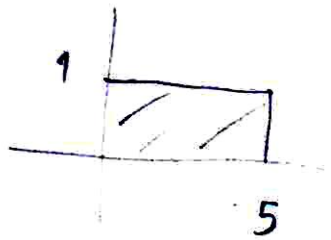
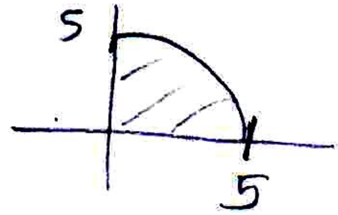
$f(c) = 2 \Rightarrow -c + 2 = 2 \Rightarrow c = 0$

34.



area $= \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = 1$

35. $\int_0^5 1 + \sqrt{25+x^2}\, dx = \int_0^5 1\, dx + \int_0^5 \sqrt{25+x^2}\, dx$

$=$  $+$ 

$= 1 \cdot 5 + \frac{1}{2}\pi(5)^2 = 5 + \frac{25\pi}{2}$

36. (a) $F(x) = -\int_0^x \sin(t^2)\,dt + \int_0^{e^x} \sin(t^2)\,dt$

$F'(x) = -\sin(x^2) + \sin([e^x]^2)\cdot(e^x)'$

$= -\sin(x^2) + \sin(e^{2x})\cdot e^x$

(b) $G(x) = x \cdot \int_1^{x^2} \sqrt{t^2-1}\,dt$

$G'(x) = x \cdot \sqrt{(x^2)^2-1}\cdot(x^2)' + \int_1^{x^2} \sqrt{t^2-1}\,dt \cdot 1$

$= 2x^2\sqrt{x^4-1} + \int_1^{x^2} \sqrt{t^2-1}\,dt$

37. $av(f) = \frac{1}{2-0}\int_0^2 3x^2\,dx = \frac{1}{2}\left[x^3\right]_0^2 = 4$

$f(c) = 4 \Rightarrow 3c^2 = 4 \Rightarrow c^2 = \frac{4}{3}$

$\Rightarrow c = \pm\frac{2}{\sqrt{3}}$

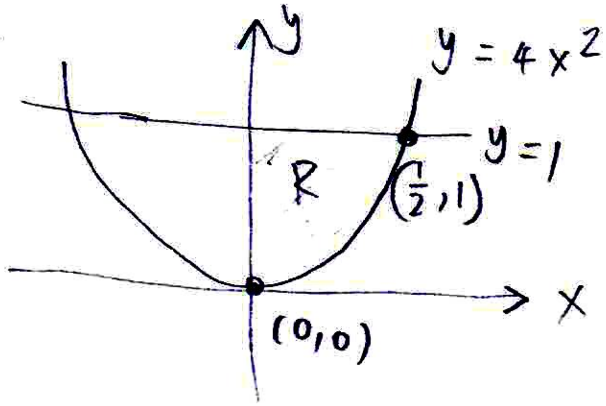
But $c \in [0,2]$, so $c = \frac{2}{\sqrt{3}}$.

38. (a) $-\dfrac{44991}{70}$

(b) $\dfrac{e^3}{36} + \dfrac{1}{72}$

(c) $\dfrac{\sqrt{2}}{6}$

39.



$$A(R) = \int_0^{\frac{1}{2}} 1 - 4x^2 \, dx$$

$$= \frac{1}{3}$$

40. (a) 1

(b) 1

(c) Undefined

41. $\dfrac{2}{s^3}$