1. (a)

Let $u = 7x - 1 \Rightarrow du = 7\,dx \Rightarrow \frac{1}{7}du = dx$

$$\int 7\sqrt{7x-1}\,dx = \int 7(7x-1)^{1/2}\,dx = \int 7u^{1/2}\,\tfrac{1}{7}\,du = \int u^{1/2}\,du = \tfrac{2}{3}u^{3/2} + C = \tfrac{2}{3}(7x-1)^{3/2} + C$$

1. (b)

Let $u = x^4 + 1 \Rightarrow du = 4x^3\,dx \Rightarrow \frac{1}{4}du = x^3\,dx$

$$\int \frac{4x^3}{(x^4+1)^2}\,dx = \int 4x^3(x^4+1)^{-2}\,dx = \int 4u^{-2}\,\tfrac{1}{4}du = \int u^{-2}du = -u^{-1} + C = \frac{-1}{x^4+1} + C$$

1. (c)

Let $u = 1 + \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$

$$\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx = \int (1+\sqrt{x})^{1/3}\,\tfrac{1}{\sqrt{x}}\,dx = \int u^{1/3}\,2\,du = 2\int u^{1/3}du = 2\cdot\tfrac{3}{4}u^{4/3} + C = \tfrac{3}{2}(1+\sqrt{x})^{4/3} + C$$

1. (d)

Let $u = 2x^2 \Rightarrow du = 4x\,dx \Rightarrow \frac{1}{4}du = x\,dx$

$$\int x\sin(2x^2)\,dx = \int \frac{1}{4}\sin u\,du = -\frac{1}{4}\cos u + C = -\frac{1}{4}\cos 2x^2 + C$$

1. (e)

Let $u = 1 - \cos\frac{t}{2} \Rightarrow du = \frac{1}{2}\sin\frac{t}{2}\,dt \Rightarrow 2\,du = \sin\frac{t}{2}\,dt$

$$\int\left(1-\cos\tfrac{t}{2}\right)^2\left(\sin\tfrac{t}{2}\right)dt = \int 2u^2du = \tfrac{2}{3}u^3 + C = \tfrac{2}{3}\left(1-\cos\tfrac{t}{2}\right)^3 + C$$

1. (f)

Let $u = y^4 + 4y^2 + 1 \Rightarrow du = (4y^3 + 8y)\,dy \Rightarrow 3\,du = 12\,(y^3+2y)\,dy$ $\int 12(y^4+4y^2+1)^2(y^3+2y)\,dy$
$= \int 3u^2\,du = u^3 + C = (y^4+4y^2+1)^3 + C$

1. (g)

Let $u = -\frac{1}{x} \Rightarrow du = \frac{1}{x^2}dx$

$$\int \tfrac{1}{x^2}\cos^2\left(\tfrac{1}{x}\right)dx = \int \cos^2(-u)\,du = \int \cos^2(u)\,du = \left(\tfrac{u}{2} + \tfrac{1}{4}\sin 2u\right) + C = -\tfrac{1}{2x} + \tfrac{1}{4}\sin\left(-\tfrac{2}{x}\right) + C = -\tfrac{1}{2x} - \tfrac{1}{4}\sin\left(\tfrac{2}{x}\right) + C$$

1. (h)

(a) Let $u = 5x + 8 \Rightarrow du = 5\ dx \Rightarrow \frac{1}{5} du = dx$

$$\int \frac{dx}{\sqrt{5x+8}} = \int \frac{1}{5}\left(\frac{1}{\sqrt{u}}\right) du = \frac{1}{5}\int u^{-1/2} du = \frac{1}{5}(2u^{1/2}) + C = \frac{2}{5}u^{1/2} + C = \frac{2}{5}\sqrt{5x+8} + C$$

(b) Let $u = \sqrt{5x+8} \Rightarrow du = \frac{1}{2}(5x+8)^{-1/2}(5)\ dx \Rightarrow \frac{2}{5} du = \frac{dx}{\sqrt{5x+8}}$

$$\int \frac{dx}{\sqrt{5x+8}} = \int \frac{2}{5}\ du = \frac{2}{5}u + C = \frac{2}{5}\sqrt{5x+8} + C$$

2. (a)

(a) Let $u = \tan x \Rightarrow du = \sec^2 x\ dx$; $v = u^3 \Rightarrow dv = 3u^2\ du \Rightarrow 6dv = 18u^2\ du$; $w = 2 + v \Rightarrow dw = dv$

$$\int \frac{18\tan^2 x \sec^2 x}{(2+\tan^3 x)^2} dx = \int \frac{18u^2}{(2+u^3)^2} du = \int \frac{6\,dv}{(2+v)^2} = \int \frac{6\,dw}{w^2} = 6\int w^{-2} dw = -6w^{-1} + C = -\frac{6}{2+v} + C$$
$$= -\frac{6}{2+u^3} + C = -\frac{6}{2+\tan^3 x} + C$$

(b) Let $u = \tan^3 x \Rightarrow du = 3\tan^2 x \sec^2 x\, dx \Rightarrow 6\,du = 18\tan^2 x \sec^2 x\, dx$; $v = 2 + u \Rightarrow dv = du$

$$\int \frac{18\tan^2 x \sec^2 x}{(2+\tan^3 x)^2} dx = \int \frac{6\,du}{(2+u)^2} = \int \frac{6\,dv}{v^2} = -\frac{6}{v} + C = -\frac{6}{2+u} + C = -\frac{6}{2+\tan^3 x} + C$$

(c) Let $u = 2 + \tan^3 x \Rightarrow du = 3\tan^2 x \sec^2 x\, dx \Rightarrow 6\,du = 18\tan^2 x \sec^2 x\, dx$

$$\int \frac{18\tan^2 x \sec^2 x}{(2+\tan^3 x)^2} dx = \int \frac{6\,du}{u^2} = -\frac{6}{u} + C = -\frac{6}{2+\tan^3 x} + C$$

2. (b)

(a) Let $u = x - 1 \Rightarrow du = dx$; $v = \sin u \Rightarrow dv = \cos u\ du$; $w = 1 + v^2 \Rightarrow dw = 2v\ dv \Rightarrow \frac{1}{2} dw = v\ dv$

$$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int \sqrt{1+\sin^2 u}\,\sin u\,\cos u\ du = \int v\sqrt{1+v^2}\ dv$$
$$= \int \frac{1}{2}\sqrt{w}\,dw = \frac{1}{3}w^{3/2} + C = \frac{1}{3}(1+v^2)^{3/2} + C = \frac{1}{3}(1+\sin^2 u)^{3/2} + C = \frac{1}{3}(1+\sin^2(x-1))^{3/2} + C$$

(b) Let $u = \sin(x-1) \Rightarrow du = \cos(x-1)\ dx$; $v = 1 + u^2 \Rightarrow dv = 2u\ du \Rightarrow \frac{1}{2} dv = u\ du$

$$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int u\sqrt{1+u^2}\,du = \int \frac{1}{2}\sqrt{v}\ dv = \int \frac{1}{2}\,v^{1/2} dv$$
$$= \left(\frac{1}{2}\left(\frac{2}{3}\right)v^{3/2}\right) + C = \frac{1}{3}v^{3/2} + C = \frac{1}{3}(1+u^2)^{3/2} + C = \frac{1}{3}(1+\sin^2(x-1))^{3/2} + C$$

(c) Let $u = 1 + \sin^2(x-1) \Rightarrow du = 2\sin(x-1)\cos(x-1)\,dx \Rightarrow \frac{1}{2} du = \sin(x-1)\cos(x-1)\,dx$

$$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int \frac{1}{2}\sqrt{u}\,du = \int \frac{1}{2}\,u^{1/2} du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C = \frac{1}{3}(1+\sin^2(x-1))^{3/2} + C$$

3.

Let $u = 2t \Rightarrow du = 2\,dt \Rightarrow 3\,du = 6dt$

$s = \int 6\sin 2t\ dt = \int (\sin u)(3\ du) = -3\cos u + C = -3\cos 2t + C$;

at $t = 0$ and $s = 0$ we have $0 = -3\cos 0 + C \Rightarrow C = 3 \Rightarrow s = 3 - 3\cos 2t \Rightarrow s\left(\frac{\pi}{2}\right) = 3 - 3\cos(\pi) = 6\ m$

4.

Let $u = \pi t \Rightarrow du = \pi dt \Rightarrow \pi\, du = \pi^2 dt$

$v = \int \pi^2 \cos \pi t\, dt = \int (\cos u)(\pi\, du) = \pi \sin u + C_1 = \pi \sin(\pi t) + C_1$;

at $t = 0$ and $v = 8$ we have $8 = \pi(0) + C_1 \Rightarrow C_1 = 8 \Rightarrow v = \frac{ds}{dt} = \pi \sin(\pi t) + 8 \Rightarrow s = \int (\pi \sin(\pi t) + 8)\, dt$

$= \int \sin u\, du + 8t + C_2 = -\cos(\pi t) + 8t + C_2$; at $t = 0$ and $s = 0$ we have $0 = -1 + C_2 \Rightarrow C_2 = 1$

$\Rightarrow s = 8t - \cos(\pi t) + 1 \Rightarrow s(1) = 8 - \cos \pi + 1 = 10\ m$

5.

All three integrations are correct. In each case, the derivative of the function on the right is the integrand on the left, and each formula has an arbitrary constant for generating the remaining antiderivatives. Moreover, $\sin^2 x + C_1 = 1 - \cos^2 x + C_1 \Rightarrow C_2 = 1 + C_1$; also $-\cos^2 x + C_2 = -\frac{\cos 2x}{2} - \frac{1}{2} + C_2 \Rightarrow C_3 = C_2 - \frac{1}{2} = C_1 + \frac{1}{2}$.

6. (a)

$$u = \ln x,\ du = \frac{1}{x}\,dx;\ dv = \frac{1}{x^2}\,dx,\ v = -\frac{1}{x}\ ;$$

$$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

6. (b)

$$u = x^2,\ du = 2x\,dx;\ dv = \sqrt{x^2+1}\,x\,dx,\ v = \tfrac{1}{3}(x^2+1)^{3/2}\ ;$$

$$\int x^3\sqrt{x^2+1}\,dx = \tfrac{1}{3}x^2(x^2+1)^{3/2} - \tfrac{1}{3}\int (x^2+1)^{3/2}2x\,dx = \tfrac{1}{3}x^2(x^2+1)^{3/2} - \tfrac{2}{15}(x^2+1)^{5/2} + C$$

6. (c)

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx\ \left[\text{Let } u = \sqrt{x},\ du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2du = \frac{1}{\sqrt{x}}\,dx\right] \rightarrow \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$$

6. (d)

$$\int \sqrt{x}\,e^{\sqrt{x}}\,dx;\ \begin{bmatrix} y = \sqrt{x} \\ dy = \frac{1}{2\sqrt{x}}\,dx \\ dx = 2y\,dy \end{bmatrix} \rightarrow \int y\,e^y\,2y\,dy = \int 2y^2\,e^y\,dy;$$

		e^y
$2y^2$	$\xrightarrow{(+)}$	e^y
$4y$	$\xrightarrow{(-)}$	e^y
4	$\xrightarrow{(+)}$	e^y
0		

$$\int 2y^2\,e^y\,dy = 2y^2e^y - 4y\,e^y + 4\,e^y + C = 2x\,e^{\sqrt{x}} - 4\sqrt{x}\,e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$$

6. (e)

$$\int \cos\sqrt{x}\,dx;\ \begin{bmatrix} y=\sqrt{x} \\ dy=\frac{1}{2\sqrt{x}}\,dx \\ dx=2y\,dy \end{bmatrix} \rightarrow \int \cos y\,2y\,dy = \int 2y\cos y\,dy;$$

$u = 2y,\ du = 2\,dy;\ dv = \cos y\,dy,\ v = \sin y$;

$$\int 2y\cos y\,dy = 2y\sin y - \int 2\sin y\,dy = 2y\sin y + 2\cos y + C = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$$

6. (f)

$u = \sec^{-1} t,\ du = \frac{dt}{t\sqrt{t^2-1}}$; $dv = t\,dt,\ v = \frac{t^2}{2}$;

$$\int_{2/\sqrt{3}}^{2} t\sec^{-1} t\,dt = \left[\frac{t^2}{2}\sec^{-1} t\right]_{2/\sqrt{3}}^{2} - \int_{2/\sqrt{3}}^{2}\left(\frac{t^2}{2}\right)\frac{dt}{t\sqrt{t^2-1}} = \left(2\cdot\frac{\pi}{3} - \frac{2}{3}\cdot\frac{\pi}{6}\right) - \int_{2/\sqrt{3}}^{2}\frac{t\,dt}{2\sqrt{t^2-1}}$$

$$= \frac{5\pi}{9} - \left[\frac{1}{2}\sqrt{t^2-1}\right]_{2/\sqrt{3}}^{2} = \frac{5\pi}{9} - \frac{1}{2}\left(\sqrt{3} - \sqrt{\frac{4}{3}-1}\right) = \frac{5\pi}{9} - \frac{1}{2}\left(\sqrt{3} - \frac{\sqrt{3}}{3}\right) = \frac{5\pi}{9} - \frac{\sqrt{3}}{3} = \frac{5\pi - 3\sqrt{3}}{9}$$

7.

(a) $u = x,\ du = dx;\ dv = \sin x\,dx,\ v = -\cos x;$

$$S_1 = \int_0^{\pi} x\sin x\,dx = [-x\cos x]_0^{\pi} + \int_0^{\pi}\cos x\,dx = \pi + [\sin x]_0^{\pi} = \pi$$

(b) $S_2 = -\int_{\pi}^{2\pi} x\sin x\,dx = -\left[[-x\cos x]_{\pi}^{2\pi} + \int_{\pi}^{2\pi}\cos x\,dx\right] = -\left[-3\pi + [\sin x]_{\pi}^{2\pi}\right] = 3\pi$

(c) $S_3 = \int_{2\pi}^{3\pi} x\sin x\,dx = [-x\cos x]_{2\pi}^{3\pi} + \int_{2\pi}^{3\pi}\cos x\,dx = 5\pi + [\sin x]_{2\pi}^{3\pi} = 5\pi$

(d) $S_{n+1} = (-1)^{n+1}\int_{n\pi}^{(n+1)\pi} x\sin x\,dx = (-1)^{n+1}\left[[-x\cos x]_{n\pi}^{(n+1)\pi} + [\sin x]_{n\pi}^{(n+1)\pi}\right]$

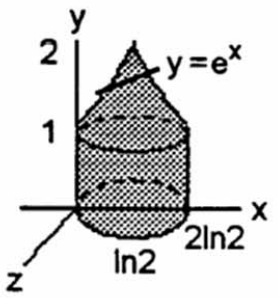
$= (-1)^{n+1}\left[-(n+1)\pi(-1)^n + n\pi(-1)^{n+1}\right] + 0 = (2n+1)\pi$

8.

$$V = \int_0^{\ln 2} 2\pi(\ln 2 - x)e^x\,dx = 2\pi\ln 2\int_0^{\ln 2} e^x\,dx - 2\pi\int_0^{\ln 2} xe^x\,dx$$

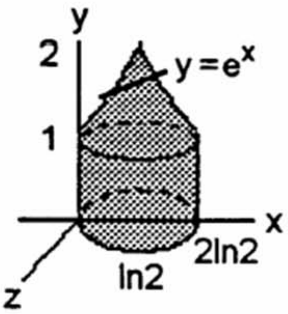
$$= (2\pi\ln 2)\left[e^x\right]_0^{\ln 2} - 2\pi\left(\left[xe^x\right]_0^{\ln 2} - \int_0^{\ln 2} e^x\,dx\right)$$

$$= 2\pi\ln 2 - 2\pi\left(2\ln 2 - \left[e^x\right]_0^{\ln 2}\right) = -2\pi\ln 2 + 2\pi = 2\pi(1 - \ln 2)$$



9.

$$V = \int_0^{\ln 2} 2\pi(\ln 2 - x)\, e^x\, dx = 2\pi \ln 2 \int_0^{\ln 2} e^x\, dx - 2\pi \int_0^{\ln 2} xe^x\, dx$$
$$= (2\pi \ln 2)\left[e^x\right]_0^{\ln 2} - 2\pi\left(\left[xe^x\right]_0^{\ln 2} - \int_0^{\ln 2} e^x\, dx\right)$$
$$= 2\pi \ln 2 - 2\pi\left(2\ln 2 - \left[e^x\right]_0^{\ln 2}\right) = -2\pi \ln 2 + 2\pi = 2\pi(1 - \ln 2)$$



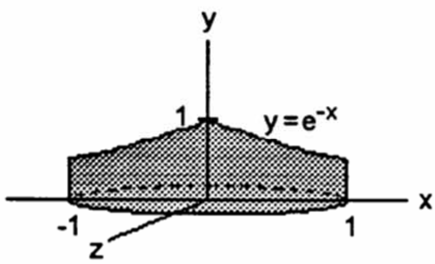
10. There is a typo.

Finding volume Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y = e^{-x}$, and the line $x = 1$

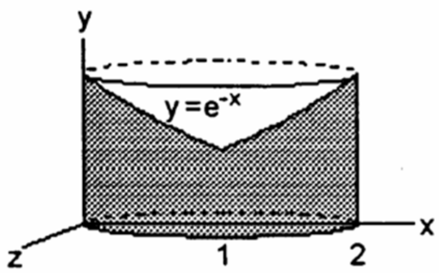
a. about the y-axis.

b. about the line $x = 1$.

(a) $V = \int_0^1 2\pi xe^{-x}\, dx = 2\pi\left(\left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\, dx\right)$
$= 2\pi\left(-\frac{1}{e} + \left[-e^{-x}\right]_0^1\right) = 2\pi\left(-\frac{1}{e} - \frac{1}{e} + 1\right)$
$= 2\pi - \frac{4\pi}{e}$

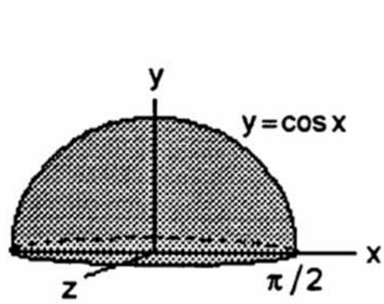


(b) $V = \int_0^1 2\pi(1 - x)e^{-x}\, dx$; $u = 1 - x$, $du = -\,dx$; $dv = e^{-x}\, dx$,
$v = -e^{-x}$; $V = 2\pi\left[\left[(1 - x)(-e^{-x})\right]_0^1 - \int_0^1 e^{-x}\, dx\right]$
$= 2\pi\left[[0 - 1(-1)] + \left[e^{-x}\right]_0^1\right] = 2\pi\left(1 + \frac{1}{e} - 1\right) = \frac{2\pi}{e}$



11.

(a) $V = \int_0^{\pi/2} 2\pi x \cos x\, dx = 2\pi\left(\left[x \sin x\right]_0^{\pi/2} - \int_0^{\pi/2} \sin x\, dx\right)$
$= 2\pi\left(\frac{\pi}{2} + \left[\cos x\right]_0^{\pi/2}\right) = 2\pi\left(\frac{\pi}{2} + 0 - 1\right) = \pi(\pi - 2)$



(b) $V = \int_0^{\pi/2} 2\pi\left(\frac{\pi}{2} - x\right)\cos x\, dx$; $u = \frac{\pi}{2} - x$, $du = -\,dx$; $dv = \cos x\, dx$, $v = \sin x$;
$V = 2\pi\left[\left(\frac{\pi}{2} - x\right)\sin x\right]_0^{\pi/2} + 2\pi\int_0^{\pi/2} \sin x\, dx = 0 + 2\pi[-\cos x]_0^{\pi/2} = 2\pi(0 + 1) = 2\pi$

12.

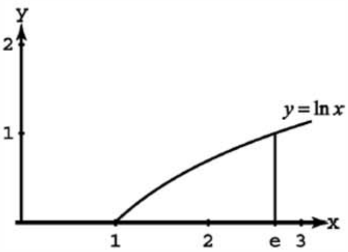
(a) $V = \int_0^{\pi} 2\pi x(x \sin x)\, dx;$

		$\sin x$
x^2	$\xrightarrow{(+)}$	$-\cos x$
$2x$	$\xrightarrow{(-)}$	$-\sin x$
2	$\xrightarrow{(+)}$	$\cos x$
0		

$$\Rightarrow V = 2\pi \int_0^{\pi} x^2 \sin x\, dx = 2\pi \left[-x^2 \cos x + 2x \sin x + 2 \cos x\right]_0^{\pi} = 2\pi\left(\pi^2 - 4\right)$$

(b) $V = \int_0^{\pi} 2\pi(\pi - x)x \sin x\, dx = 2\pi^2 \int_0^{\pi} x \sin x\, dx - 2\pi \int_0^{\pi} x^2 \sin x\, dx = 2\pi^2 \left[-x \cos x + \sin x\right]_0^{\pi} - \left(2\pi^3 - 8\pi\right)$
$= 8\pi$

13.



(a) $A = \int_1^e \ln x\, dx = \Big[x \ln x\Big]_1^e - \int_1^e dx$
$= (e \ln e - 1 \ln 1) - \Big[x\Big]_1^e = e - (e - 1) = 1$

(b) $V = \int_1^e \pi(\ln x)^2\, dx = \pi\left(\left[x\,(\ln x)^2\right]_1^e - \int_1^e 2 \ln x\, dx\right)$
$= \pi\left[\left(e(\ln e)^2 - 1(\ln 1)^2\right) - \left(\Big[2x \ln x\Big]_1^e - \int_1^e 2\, dx\right)\right]$
$= \pi\left[e - \left((2e \ln e - 2(1)\ln 1) - \Big[2x\Big]_1^e\right)\right]$
$= \pi\Big[e - (2e - (2e - 2))\Big] = \pi(e - 2)$

(c) $V = \int_1^e 2\pi(x + 2) \ln x\, dx = 2\pi \int_1^e (x + 2) \ln x\, dx = 2\pi\left(\left[\left(\tfrac{1}{2}x^2 + 2x\right)\ln x\right]_1^e - \int_1^e \left(\tfrac{1}{2}x + 2\right) dx\right)$
$= 2\pi\left(\left(\tfrac{1}{2}e^2 + 2e\right)\ln e - \left(\tfrac{1}{2} + 2\right)\ln 1 - \left[\left(\tfrac{1}{4}x^2 + 2x\right)\right]_1^e\right) = 2\pi\left(\left(\tfrac{1}{2}e^2 + 2e\right) - \left(\left(\tfrac{1}{4}e^2 + 2e\right) - \tfrac{9}{4}\right)\right) = \tfrac{\pi}{2}\left(e^2 + 9\right)$

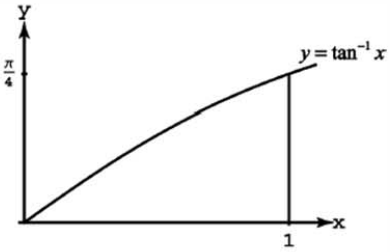
(d) $M = \int_1^e \ln x\, dx = 1$ (from part (a)); $\overline{x} = \tfrac{1}{1}\int_1^e x \ln x\, dx = \left[\tfrac{1}{2}x^2 \ln x\right]_1^e - \int_1^e \tfrac{1}{2}x\, dx = \left(\tfrac{1}{2}e^2 \ln e - \tfrac{1}{2}(1)^2 \ln 1\right) - \left[\tfrac{1}{4}x^2\right]_1^e$
$= \tfrac{1}{2}e^2 - \left(\tfrac{1}{4}e^2 - \tfrac{1}{4}(1)^2\right) = \tfrac{1}{4}\left(e^2 + 1\right)$; $\overline{y} = \tfrac{1}{1}\int_1^e \tfrac{1}{2}(\ln x)^2\, dx = \tfrac{1}{2}\left(\left[x\,(\ln x)^2\right]_1^e - \int_1^e 2 \ln x\, dx\right)$
$= \tfrac{1}{2}\left(\left(e\,(\ln e)^2 - 1 \cdot (\ln 1)^2\right) - \left(\Big[2x \ln x\Big]_1^e - \int_1^e 2\, dx\right)\right) = \tfrac{1}{2}\left(e - \left((2e \ln e - 2(1)\ln 1) - \Big[2x\Big]_1^e\right)\right)$
$= \tfrac{1}{2}(e - 2e + 2e - 2) = \tfrac{1}{2}(e - 2) \Rightarrow (\overline{x}, \overline{y}) = \left(\tfrac{e^2 + 1}{4}, \tfrac{e - 2}{2}\right)$ is the centroid.

14.

(a) $A = \int_0^1 \tan^{-1} x \, dx = \left[x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx$

$= (\tan^{-1} 1 - 0) - \frac{1}{2}\left[\ln(1 + x^2) \right]_0^1$

$= \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \ln 1) = \frac{\pi}{4} - \frac{1}{2}\ln 2$



(b) $V = \int_0^1 2\pi \, x \tan^{-1} x \, dx$

$= 2\pi \left(\left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{1+x^2} dx \right)$

$= 2\pi \left(\frac{1}{2} \tan^{-1} 1 - 0 - \frac{1}{2} \int_0^1 \left(1 - \frac{1}{1+x^2}\right) dx \right) = 2\pi \left(\frac{\pi}{8} - \frac{1}{2}\left[x - \tan^{-1} x \right]_0^1 \right) = 2\pi\left(\frac{\pi}{8} - \frac{1}{2}(1 - \tan^{-1} 1 - (0 - 0))\right)$

$= 2\pi\left(\frac{\pi}{8} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)\right) = \frac{\pi(\pi - 2)}{2}$

15. (a)

$I = \int x^n \cos x \, dx; \; [u = x^n, \, du = nx^{n-1} \, dx; \, dv = \cos x \, dx, \, v = \sin x]$

$\Rightarrow I = x^n \sin x - \int nx^{n-1} \sin x \, dx$

15. (b)

$I = \int x^n e^{ax} \, dx; \; \left[u = x^n, \, du = nx^{n-1} \, dx; \, dv = e^{ax} \, dx, \, v = \frac{1}{a} e^{ax} \right]$

$\Rightarrow I = \frac{x^n e^{ax}}{a} e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx, \; a \neq 0$

16.

$\int \sqrt{1 - x^2} \, dx; \; \left[u = \sqrt{1 - x^2}, \, du = \frac{-x}{\sqrt{1-x^2}} dx; \, dv = dx, \, v = x \right]$

$= x\sqrt{1 - x^2} - \int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1 - x^2} - \int \frac{1 - x^2 - 1}{\sqrt{1-x^2}} dx = x\sqrt{1 - x^2} - \left(\int \frac{1-x^2}{\sqrt{1-x^2}} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right)$

$= x\sqrt{1 - x^2} - \int \sqrt{1 - x^2} \, dx + \int \frac{1}{\sqrt{1-x^2}} dx$

$\Rightarrow \int \sqrt{1 - x^2} \, dx = x\sqrt{1 - x^2} + \int \frac{1}{\sqrt{1-x^2}} dx - \int \sqrt{1 - x^2} \, dx \Rightarrow 2\int \sqrt{1 - x^2} \, dx = x\sqrt{1 - x^2} + \int \frac{1}{\sqrt{1-x^2}} dx$

$\Rightarrow \int \sqrt{1 - x^2} \, dx = \frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} dx + C$

17.

$y = \ln(\sec x); \; y' = \frac{\sec x \tan x}{\sec x} = \tan x; (y')^2 = \tan^2 x; \int_0^{\pi/4} \sqrt{1 + \tan^2 x} \, dx = \int_0^{\pi/4} |\sec x| \, dx = [\ln|\sec x + \tan x|]_0^{\pi/4}$

$= \ln\left(\sqrt{2} + 1\right) - \ln(0 + 1) = \ln\left(\sqrt{2} + 1\right)$

18.

$$V = \pi\int_0^{\pi} \sin^2 x\, dx = \pi\int_0^{\pi} \frac{1-\cos 2x}{2}\, dx = \frac{\pi}{2}\int_0^{\pi} dx - \frac{\pi}{2}\int_0^{\pi} \cos 2x\, dx = \frac{\pi}{2}[x]_0^{\pi} - \frac{\pi}{4}[\sin 2x]_0^{\pi} = \frac{\pi}{2}(\pi - 0) - \frac{\pi}{4}(0-0) = \frac{\pi^2}{2}$$

19.

$$V = \int_0^{\pi/3} \pi(\sin x + \sec x)^2\, dx = \pi\int_0^{\pi/3} (\sin^2 x + 2\sin x \sec x + \sec^2 x)\, dx$$

$$= \pi\int_0^{\pi/3} \sin^2 x\, dx + \pi\int_0^{\pi/3} 2\tan x\, dx + \pi\int_0^{\pi/3} \sec^2 x\, dx = \pi\int_0^{\pi/3} \frac{1-\cos 2x}{2}\, dx + 2\pi\Big[\ln|\sec x|\Big]_0^{\pi/3} + \pi\Big[\tan x\Big]_0^{\pi/3}$$

$$= \frac{\pi}{2}\int_0^{\pi/3} dx - \frac{\pi}{2}\int_0^{\pi/3} \cos 2x\, dx + 2\pi\left(\ln|\sec \tfrac{\pi}{3}| - \ln|\sec 0|\right) + \pi\left(\tan \tfrac{\pi}{3} - \tan 0\right)$$

$$= \frac{\pi}{2}\Big[x\Big]_0^{\pi/3} - \frac{\pi}{4}\Big[\sin 2x\Big]_0^{\pi/3} + 2\pi \ln 2 + \pi\sqrt{3} = \frac{\pi}{2}\left(\frac{\pi}{3} - 0\right) - \frac{\pi}{4}\left(\sin 2\left(\frac{\pi}{3}\right) - \sin 2(0)\right) + 2\pi\ln 2 + \pi\sqrt{3}$$

$$= \frac{\pi^2}{6} - \frac{\pi\sqrt{3}}{8} + 2\pi \ln 2 + \pi\sqrt{3} = \frac{\pi\left(4\pi + 21\sqrt{3} - 48\ln 2\right)}{24}$$

20.

$$A = \int_0^3 \frac{\sqrt{9-x^2}}{3}\, dx;\ x = 3\sin\theta,\ 0 \le \theta \le \frac{\pi}{2},\ dx = 3\cos\theta\, d\theta,\ \sqrt{9-x^2} = \sqrt{9 - 9\sin^2\theta} = 3\cos\theta;$$

$$A = \int_0^{\pi/2} \frac{3\cos\theta \cdot 3\cos\theta\, d\theta}{3} = 3\int_0^{\pi/2} \cos^2\theta\, d\theta = \frac{3}{2}\left[\theta + \sin\theta\cos\theta\right]_0^{\pi/2} = \frac{3\pi}{4}$$

21.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = \pm b\sqrt{1 - \frac{x^2}{a^2}};\ A = 4\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\, dx = 4b\int_0^a \sqrt{1 - \frac{x^2}{a^2}}\, dx$$

$$\left[x = a\sin\theta,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ dx = a\cos\theta\, d\theta,\ \sqrt{1-\frac{x^2}{a^2}} = \cos\theta,\ x = 0 = a\sin\theta \Rightarrow \theta = 0,\ x = a = a\sin\theta \Rightarrow \theta = \frac{\pi}{2}\right]$$

$$4b\int_0^a \sqrt{1-\frac{x^2}{a^2}}\, dx = 4b\int_0^{\pi/2} \cos\theta\,(a\cos\theta)\, d\theta = 4ab\int_0^{\pi/2} \cos^2\theta\, d\theta = 4ab\int_0^{\pi/2} \frac{1+\cos 2\theta}{2} d\theta$$

$$= 2ab\int_0^{\pi/2} d\theta + 2ab\int_0^{\pi/2} \cos 2\theta\, d\theta = 2ab\Big[\theta\Big]_0^{\pi/2} + ab\Big[\sin 2\theta\Big]_0^{\pi/2} = 2ab\left(\frac{\pi}{2} - 0\right) + ab(\sin\pi - \sin 0) = \pi ab$$

22.

(a) Integration by parts: $u = x^2,\ du = 2x\,dx,\ dv = x\sqrt{1-x^2}\,dx,\ v = -\frac{1}{3}(1-x^2)^{3/2}$

$$\int x^3\sqrt{1-x^2}\, dx = -\frac{1}{3}x^2(1-x^2)^{3/2} + \frac{1}{3}\int (1-x^2)^{3/2}\, 2x\, dx = -\frac{1}{3}x^2(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2} + C$$

(b) Substitution: $u = 1 - x^2 \Rightarrow x^2 = 1 - u \Rightarrow du = -2x\,dx \Rightarrow -\frac{1}{2}du = x\,dx$

$$\int x^3\sqrt{1-x^2}\, dx = \int x^2\sqrt{1-x^2}\, x\, dx = -\frac{1}{2}\int (1-u)\sqrt{u}\, du = -\frac{1}{2}\int \left(\sqrt{u} - u^{3/2}\right) du = -\frac{1}{3}u^{3/2} + \frac{1}{5}u^{5/2} + C$$

$$= -\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + C$$

(c) Trig substitution: $x = \sin\theta,\ \frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ dx = \cos\theta\, d\theta,\ \sqrt{1-x^2} = \cos\theta$

$$\int x^3\sqrt{1-x^2}\, dx = \int \sin^3\theta\cos\theta\cos\theta\, d\theta = \int \sin^2\theta\cos^2\theta\sin\theta\, d\theta = \int (1-\cos^2\theta)\cos^2\theta\sin\theta\, d\theta$$

$$= \int \cos^2\theta\sin\theta\, d\theta - \int \cos^4\theta\sin\theta\, d\theta = -\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + C = -\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + C$$

23. (a)

$$\int \frac{\sqrt{x+1}}{x}\,dx\ \left[\text{Let } x+1=u^2 \Rightarrow dx = 2u\,du\right] \to \int \frac{u}{u^2-1}\,2u\,du = \int \frac{2u^2}{u^2-1}\,du = \int \left(2+\frac{2}{u^2-1}\right)du$$
$$= 2\int du + \int \frac{2}{u^2-1}du;\ \frac{2}{u^2-1} = \frac{A}{u+1}+\frac{B}{u-1} \Rightarrow 2 = A(u-1)+B(u+1) = (A+B)u - A + B \Rightarrow A+B=0,$$
$$-A+B=2 \Rightarrow B=1 \Rightarrow A=-1;\ 2\int du + \int \frac{2}{u^2-1}du = 2u + \int\left(\frac{-1}{u+1}+\frac{1}{u-1}\right)du = 2u - \int \frac{1}{u+1}du + \int \frac{1}{u-1}du$$
$$= 2u - \ln|u+1| + \ln|u-1| + C = 2\sqrt{x+1} + \ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C$$

23. (b)

$$\int \frac{1}{x(x^4+1)}dx = \int \frac{x^3}{x^4(x^4+1)}dx\ \left[\text{Let } u = x^4 \Rightarrow du = 4x^3\,dx\right] \to \frac{1}{4}\int \frac{1}{u(u+1)}du;\ \frac{1}{u(u+1)} = \frac{A}{u}+\frac{B}{u+1}$$
$$\Rightarrow 1 = A(u+1)+Bu = (A+B)u + A \Rightarrow A = 1 \Rightarrow B = -1;\ \frac{1}{4}\int \frac{1}{u(u+1)}du = \frac{1}{4}\int\left(\frac{1}{u}-\frac{1}{u+1}\right)du$$
$$= \frac{1}{4}\int \frac{1}{u}du - \frac{1}{4}\int \frac{1}{u+1}du = \frac{1}{4}\ln|u| - \frac{1}{4}\ln|u+1| + C = \frac{1}{4}\ln\left(\frac{x^4}{x^4+1}\right) + C$$

24. (a)

$$(t^2-3t+2)\,\frac{dx}{dt} = 1;\ x = \int \frac{dt}{t^2-3t+2} = \int \frac{dt}{t-2} - \int \frac{dt}{t-1} = \ln\left|\frac{t-2}{t-1}\right| + C;\ \frac{t-2}{t-1} = Ce^x;\ t=3 \text{ and } x=0$$
$$\Rightarrow\ \frac{1}{2} = C\ \Rightarrow\ \frac{t-2}{t-1} = \frac{1}{2}e^x\ \Rightarrow\ x = \ln\left|2\left(\frac{t-2}{t-1}\right)\right| = \ln|t-2| - \ln|t-1| + \ln 2$$

24. (b)

$$(t^2+2t)\,\frac{dx}{dt} = 2x+2;\ \frac{1}{2}\int \frac{dx}{x+1} = \int \frac{dt}{t^2+2t}\ \Rightarrow\ \frac{1}{2}\ln|x+1| = \frac{1}{2}\int \frac{dt}{t} - \frac{1}{2}\int \frac{dt}{t+2}\ \Rightarrow\ \ln|x+1| = \ln\left|\frac{t}{t+2}\right| + C;$$
$$t=1 \text{ and } x=1\ \Rightarrow\ \ln 2 = \ln\frac{1}{3} + C\ \Rightarrow\ C = \ln 2 + \ln 3 = \ln 6\ \Rightarrow\ \ln|x+1| = \ln 6\left|\frac{t}{t+2}\right|\ \Rightarrow\ x+1 = \frac{6t}{t+2}$$
$$\Rightarrow\ x = \frac{6t}{t+2} - 1,\ t > 0$$

25.

The length of the curve $y = \sin\left(\frac{3\pi}{20}x\right)$ from 0 to 20 is: $L = \int_0^{20}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx;\ \frac{dy}{dx} = \frac{3\pi}{20}\cos\left(\frac{3\pi}{20}x\right)\ \Rightarrow\ \left(\frac{dy}{dx}\right)^2$ $= \frac{9\pi^2}{400}\cos^2\left(\frac{3\pi}{20}x\right)\ \Rightarrow\ L = \int_0^{20}\sqrt{1+\frac{9\pi^2}{400}\cos^2\left(\frac{3\pi}{20}x\right)}\,dx$. Using numerical integration we find $L \approx 21.07$ in

26.

First, we'll find the length of the cosine curve: $L = \int_{-25}^{25}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx;\ \frac{dy}{dx} = -\frac{25\pi}{50}\sin\left(\frac{\pi x}{50}\right)$ $\Rightarrow\ \left(\frac{dy}{dx}\right)^2 = \frac{\pi^2}{4}\sin^2\left(\frac{\pi x}{50}\right)\ \Rightarrow\ L = \int_{-25}^{25}\sqrt{1+\frac{\pi^2}{4}\sin^2\left(\frac{\pi x}{50}\right)}\,dx$. Using a numerical integrator we find $L \approx 73.1848$ ft. Surface area is: $A = \text{length} \cdot \text{width} \approx (73.1848)(300) = 21{,}955.44$ ft.
Cost $= 1.75A = (1.75)(21{,}955.44) = \$38{,}422.02$. Answers may vary slightly, depending on the numerical integration used.

27.

$y = \left(1 - x^{2/3}\right)^{3/2}, \frac{\sqrt{2}}{4} \le x \le 1 \Rightarrow \frac{dy}{dx} = \frac{3}{2}\left(1 - x^{2/3}\right)^{1/2}\left(-\frac{2}{3}x^{-1/3}\right) = -\frac{\left(1 - x^{2/3}\right)^{1/2}}{x^{1/3}} \Rightarrow L = \int_{\sqrt{2}/4}^{1} \sqrt{1 + \left[-\frac{\left(1 - x^{2/3}\right)^{1/2}}{x^{1/3}}\right]^2}\, dx$

$= \int_{\sqrt{2}/4}^{1} \sqrt{1 + \frac{1 - x^{2/3}}{x^{2/3}}}\, dx = \int_{\sqrt{2}/4}^{1} \sqrt{1 + \frac{1}{x^{2/3}} - 1}\, dx = \int_{\sqrt{2}/4}^{1} \sqrt{\frac{1}{x^{2/3}}}\, dx = \int_{\sqrt{2}/4}^{1} \frac{1}{x^{1/3}}\, dx = \int_{\sqrt{2}/4}^{1} x^{-1/3}\, dx = \frac{3}{2}\left[x^{2/3}\right]_{\sqrt{2}/4}^{1}$

$= \frac{3}{2}(1)^{2/3} - \frac{3}{2}\left(\frac{\sqrt{2}}{4}\right)^{2/3} = \frac{3}{2} - \frac{3}{2}\left(\frac{1}{2}\right) = \frac{3}{4} \Rightarrow$ total length $= 8\left(\frac{3}{4}\right) = 6$

28.

Let (x_1, y_1) and (x_2, y_2), with $x_2 > x_1$, lie on $y = mx + b$, where $m = \frac{y_2 - y_1}{x_2 - x_1}$, then $\frac{dy}{dx} = m \Rightarrow L = \int_{x_1}^{x_2} \sqrt{1 + m^2}\, dx$

$= \sqrt{1 + m^2}\, [x]_{x_1}^{x_2} = \sqrt{1 + m^2}(x_2 - x_1) = \sqrt{1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2}(x_2 - x_1) = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{(x_2 - x_1)^2}}(x_2 - x_1)$

$\frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{(x_2 - x_1)}(x_2 - x_1) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$

29.

$R(y) = \sqrt{256 - y^2} \Rightarrow V = \int_{-16}^{-7} \pi \left[R(y)\right]^2\, dy = \pi \int_{-16}^{-7} \left(256 - y^2\right) dy = \pi \left[256y - \frac{y^3}{3}\right]_{-16}^{-7}$

$= \pi \left[(256)(-7) + \frac{7^3}{3} - \left((256)(-16) + \frac{16^3}{3}\right)\right] = \pi \left(\frac{7^3}{3} + 256(16 - 7) - \frac{16^3}{3}\right) = 1053\pi \text{ cm}^3 \approx 3308 \text{ cm}^3$