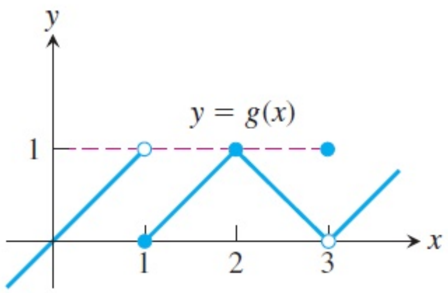
Limit & Continuity of Function

Limits from Graphs

1. For the function $g(x)$ graphed here, find the following limits or explain why they do not exist.



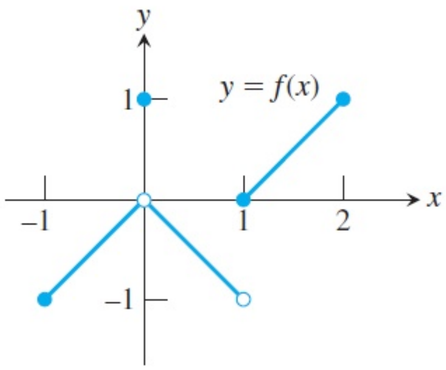
1.1 $\lim_{x\to 1} g(x)$

1.2 $\lim_{x\to 2} g(x)$

1.3 $\lim_{x\to 3} g(x)$

1.4 $\lim_{x\to 2.5} g(x)$

2. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



2.1 $\lim_{x\to 0} f(x)$ exists.

2.2 $\lim_{x\to 0} f(x) = 0$

2.3 $\lim_{x\to 0} f(x) = 1$

2.4 $\lim_{x\to 1} f(x) = 1$

2.5 $\lim_{x\to 1} f(x) = 0$

2.6 $\lim_{x\to x_0} f(x)$ exists at $\forall x_0 \in (-1, 1)$.

2.7 $\lim_{x\to 1} f(x)$ does not exist.

Existence of Limits

1. Explain why the limits do not exist.

1.1 $\lim_{x\to 0} \dfrac{x}{|x|}$

1.2 $\lim_{x\to 1} \dfrac{1}{x-1}$

2. Suppose that a function $f(x)$ is defined for all real values of x except $x = x_0$. Can anything be said about the existence of $\lim_{x\to x_0} f(x)$? Give reasons for your answer.

3. Suppose that a function $f(x)$ is defined for all x in $[-1, 1]$. Can anything be said about the existence of $\lim_{x\to 0} f(x)$? Give reasons for your answer.

4. If $\lim_{x\to 1} f(x) = 5$, must f be defined at $x = 1$? If it is, must $f(1) = 5$? Can we conclude anything about the values of f at $x = 1$? Explain.

5. If $f(1) = 5$, must $\lim_{x\to 1} f(x)$ exist? If it does, then must $\lim_{x\to 1} f(x) = 5$? Can we conclude anything about $\lim_{x\to 1} f(x)$? Explain.

Using Limit Rules

1. Suppose $\lim_{x\to c} f(x) = 5$ and $\lim_{x\to c} g(x) = -2$. Find

   1.1 $\lim_{x\to c} f(x)g(x)$

   1.2 $\lim_{x\to c} \dfrac{f(x)}{g(x)}$

   1.3 $\lim_{x\to c}(f(x) + 3g(x))$

   1.4 $\lim_{x\to c} \dfrac{f(x)}{f(x) - g(x)}$

Limits of Average Rates of Change

Because of their connection with secant lines, tangents, and instantaneous rates, limits of the form

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

occur frequently in calculus. Evaluate this limit for the given value of x and function f.

1. $f(x) = x^2, \quad x = 1$
2. $f(x) = x^2, \quad x = -2$
3. $f(x) = 3x - 4, \quad x = 2$
4. $f(x) = 1/x, \quad x = -2$
5. $f(x) = \sqrt{x}, \quad x = 7$
6. $f(x) = \sqrt{3x+1}, \quad x = 0$

Theory & Examples

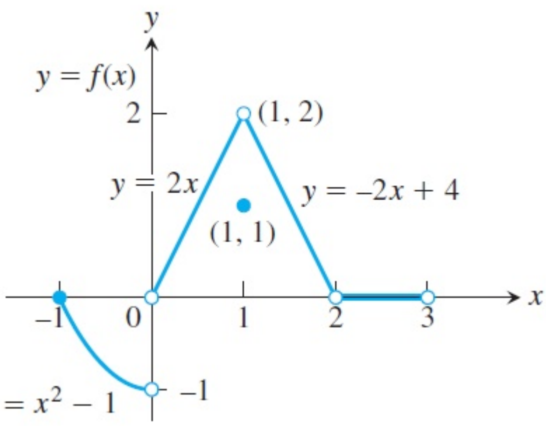
1. Once you know $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ at an interior point of the domain of f, do you then know $\lim_{x\to a} f(x)$? Give reasons for your answer.

2. If you know that $\lim_{x\to c} f(x)$ exists, can you find its value by calculating $\lim_{x\to c^+} f(x)$? Give reasons for your answer.

Continuity from Graphs

Consider the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



graphied in the accompanying figure.

1. Does $f(-1)$ exist ?
   Does $\lim_{x\to -1^+} f(x)$ exist?
   Does $\lim_{x\to -1^+} f(x) = f(-1)$?
   Is f continuous at $x = -1$?

2. Does $f(1)$ exist ?
   Does $\lim_{x\to 1} f(x)$ exist?
   Does $\lim_{x\to 1} f(x) = f(1)$?
   Is f continuous at $x = 1$?

3. Is f defined at $x = 2$?
   Is f continuous at $x = 2$

4. At what values of x is f continuous?

5. What value should be assigned to $f(2)$ to make the extended function continous at $x = 2$?

6. To what new value should $f(1)$ be changed to remove the discontiuity?