

The chain rule

Exercise

91. Find dy/dx if $y = x$ by using the Chain Rule with y as a composite of
- $y = (u/5) + 7$ and $u = 5x - 35$
 - $y = 1 + (1/u)$ and $u = 1/(x - 1)$.
92. Find dy/dx if $y = x^{3/2}$ by using the Chain Rule with y as a composite of
- $y = u^3$ and $u = \sqrt{x}$
 - $y = \sqrt{u}$ and $u = x^3$.
93. Find the tangent to $y = ((x - 1)/(x + 1))^2$ at $x = 0$.
94. Find the tangent to $y = \sqrt{x^2 - x + 7}$ at $x = 2$.
62. Air is being pumped into a spherical weather balloon. At any time t , the volume of the balloon is $V(t)$ and its radius is $r(t)$.
- What do the derivatives dV/dr and dV/dt represent?
 - Express dV/dt in terms of dr/dt .

97. **Running machinery too fast** Suppose that a piston is moving straight up and down and that its position at time t sec is

$$s = A \cos(2\pi bt),$$

with A and b positive. The value of A is the amplitude of the motion, and b is the frequency (number of times the piston moves up and down each second). What effect does doubling the frequency have on the piston's velocity, acceleration, and jerk? (Once you find out, you will know why some machinery breaks when you run it too fast.)

99. **Particle motion** The position of a particle moving along a coordinate line is $s = \sqrt{1 + 4t}$, with s in meters and t in seconds. Find the particle's velocity and acceleration at $t = 6$ sec.
100. **Constant acceleration** Suppose that the velocity of a falling body is $v = k\sqrt{s}$ m/sec (k a constant) at the instant the body has fallen s m from its starting point. Show that the body's acceleration is constant.
101. **Falling meteorite** The velocity of a heavy meteorite entering Earth's atmosphere is inversely proportional to \sqrt{s} when it is s km from Earth's center. Show that the meteorite's acceleration is inversely proportional to s^2 .

Implicit differentiations

Exercise

In Exercises 21–26, use implicit differentiation to find dy/dx and then d^2y/dx^2 .

21. $x^2 + y^2 = 1$

22. $x^{2/3} + y^{2/3} = 1$

23. $y^2 = e^x + 2x$

24. $y^2 - 2x = 1 - 2y$

25. $2\sqrt{y} = x - y$

26. $xy + y^2 = 1$

27. If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point $(2, 2)$.

28. If $xy + y^2 = 1$, find the value of d^2y/dx^2 at the point $(0, -1)$.

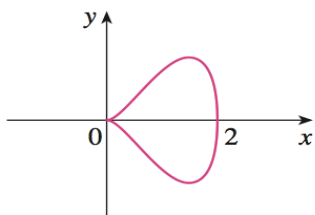
In Exercises 29 and 30, find the slope of the curve at the given points.

29. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$

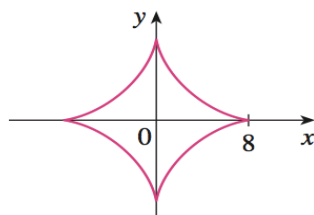
30. $(x^2 + y^2)^2 = (x - y)^2$ at $(1, 0)$ and $(1, -1)$

13–18 ■ Find an equation of the tangent line to the curve at the given point.

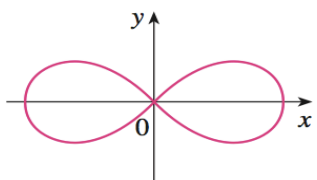
15. $y^2 = x^3(2 - x)$
 $(1, 1)$
 (piriform)



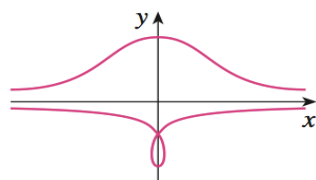
16. $x^{2/3} + y^{2/3} = 4$
 $(-3\sqrt{3}, 1)$
 (astroid)



17. $2(x^2 + y^2)^2 = 25(x^2 - y^2)$
 $(3, 1)$
 (lemniscate)



18. $x^2y^2 = (y + 1)^2(4 - y^2)$
 $(0, -2)$
 (conchoid of Nicomedes)



Higher order derivatives

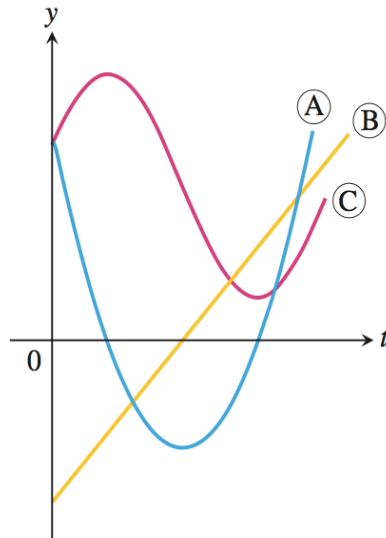
Exercise

EXAMPLE 3 Figure 3.17 shows the free fall of a heavy ball bearing released from rest at time $t = 0$ sec.

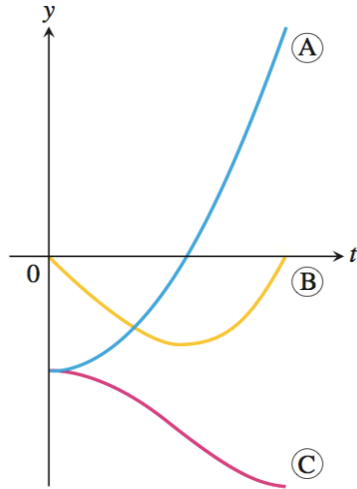
- (a) How many meters does the ball fall in the first 2 sec?
- (b) What is its velocity, speed, and acceleration when $t = 2$?

EXAMPLE 4 A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec (about 109 mph) (Figure 3.18a). It reaches a height of $s = 160t - 16t^2$ ft after t sec.

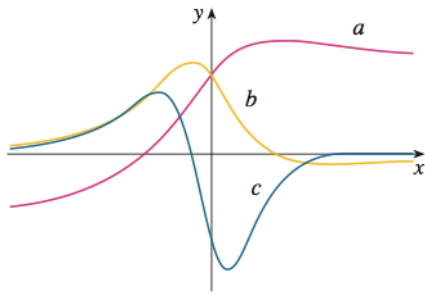
- (a) How high does the rock go?
 - (b) What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?
 - (c) What is the acceleration of the rock at any time t during its flight (after the blast)?
 - (d) When does the rock hit the ground again?
21. The graphs in the accompanying figure show the position s , velocity $v = ds/dt$, and acceleration $a = d^2s/dt^2$ of a body moving along a coordinate line as functions of time t . Which graph is which? Give reasons for your answers.



22. The graphs in the accompanying figure show the position s , the velocity $v = ds/dt$, and the acceleration $a = d^2s/dt^2$ of a body moving along the coordinate line as functions of time t . Which graph is which? Give reasons for your answers.



35. The figure shows the graphs of f , f' , and f'' . Identify each curve, and explain your choices.



36. The figure shows graphs of f , f' , f'' , and f''' . Identify each curve, and explain your choices.

