Sums of rectangles evaluated at midpoints can be represented and evaluated by this set of commands.

```
Clear[x, f, a, b, n]
{a, b}={0, π}; n =10; dx = (b – a)/n;
f = Sin[x]^2;
xvals =Table[N[x], {x, a + dx/2, b – dx/2, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1, 0},{#1, #3},{#2, #3},{#2, 0}}]&,{xvals, – dx/2, xvals + dx/2, yvals}];
Plot[f, {x, a, b},Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1, Length[yvals]}]//N
```

5.4 THE FUNDAMENTAL THEOREM OF CALCULUS

1. $\int_0^2 x(x-3)\,dx = \int_0^2 (x^2-3x)\,dx = \left[\frac{x^3}{3}-\frac{3x^2}{2}\right]_0^2 = \left(\frac{(2)^3}{3}-\frac{3(2)^2}{2}\right)-\left(\frac{(0)^3}{3}-\frac{3(0)^2}{2}\right) = -\frac{10}{3}$

2. $\int_{-1}^1 \left(x^2-2x+3\right)dx = \left[\frac{x^3}{3}-x^2+3x\right]_{-1}^1 = \left(\frac{(1)^3}{3}-(1)^2+3(1)\right)-\left(\frac{(-1)^3}{3}-(-1)^2+3(-1)\right) = \frac{20}{3}$

3. $\int_{-2}^2 \frac{3}{(x+3)^4}dx = -\frac{1}{(x+3)^3}\Big]_{-2}^2 = \left(-\frac{1}{(5)^3}-\left(-\frac{1}{(1)^3}\right)\right) = 1-\frac{1}{125} = \frac{124}{125}$

4. $\int_{-1}^1 x^{299}\,dx = \frac{x^{300}}{300}\Big]_{-1}^1 = \frac{1}{300}\left((1)^{300}-(-1)^{300}\right) = \frac{1}{300}(1-1) = 0$

5. $\int_1^4 \left(3x^2-\frac{x^3}{4}\right)dx = \left[x^3-\frac{x^4}{16}\right]_1^4 = \left(\left(4^3-\frac{4^4}{16}\right)-\left(1^3-\frac{1^4}{16}\right)\right) = \left(64-16-1+\frac{1}{16}\right) = \frac{753}{16}$

6. $\int_{-2}^3 \left(x^3-2x+3\right)dx = \left[\frac{x^4}{4}-x^2+3x\right]_1^4$

$= \left(\frac{3^4}{4}-3^2+3(3)\right)-\left(\frac{(-2)^4}{4}-(-2)^2+3(-2)\right) = \frac{81}{4}+6 = \frac{105}{4}$

7. $\int_0^1 \left(x^2+\sqrt{x}\right)dx = \left[\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right]_0^1 = \left(\frac{1}{3}+\frac{2}{3}\right)-0 = 1$

8. $\int_1^{32} x^{-6/5}dx = \left[-5x^{-1/5}\right]_1^{32} = \left(-\frac{5}{2}\right)-(-5) = \frac{5}{2}$

9. $\int_0^{\pi/3} 2\sec^2 x\,dx = [2\tan x]_0^{\pi/3} = \left(2\tan\left(\frac{\pi}{3}\right)\right)-(2\tan 0) = 2\sqrt{3}-0 = 2\sqrt{3}$

[redacted]

10. $\int_0^{\pi}(1+\cos x)\,dx = [x+\sin x]_0^{\pi} = (\pi+\sin\pi)-(0+\sin 0) = \pi$

11. $\int_{\pi/4}^{3\pi/4}\csc\theta\cot\theta\,d\theta = [-\csc\theta]_{\pi/4}^{3\pi/4} = \left(-\csc\left(\frac{3\pi}{4}\right)\right)-\left(-\csc\left(\frac{\pi}{4}\right)\right) = -\sqrt{2}-\left(-\sqrt{2}\right) = 0$

12. $\int_0^{\pi/3} 4\frac{\sin u}{\cos^2 u}\,du = \frac{4}{\cos u}\Big]_0^{\pi/3} = \left(\frac{4}{(1/2)}-\frac{4}{1}\right) = 4$

13. $\int_{\pi/2}^{0}\frac{1+\cos 2t}{2}\,dt = \int_{\pi/2}^{0}\left(\frac{1}{2}+\frac{1}{2}\cos 2t\right)dt = \left[\frac{1}{2}t+\frac{1}{4}\sin 2t\right]_{\pi/2}^{0} = \left(\frac{1}{2}(0)+\frac{1}{4}\sin 2(0)\right)-\left(\frac{1}{2}\left(\frac{\pi}{2}\right)+\frac{1}{4}\sin 2\left(\frac{\pi}{2}\right)\right) = -\frac{\pi}{4}$

14. $\int_{-\pi/3}^{\pi/3}\sin^2 t\,dt$ Use the double angle formula $\cos 2t = 1-2\sin^2 t$ which implies that $\sin^2 t = \frac{1-\cos(2t)}{2}$.

$$\int_{-\pi/3}^{\pi/3}\sin^2 t\,dt = \int_{-\pi/3}^{\pi/3}\frac{1-\cos 2t}{2}\,dt = \left[\frac{t}{2}-\frac{\sin 2t}{4}\right]_{-\pi/3}^{\pi/3}$$
$$= \left(\frac{\pi}{6}-\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right)-\left(-\frac{\pi}{6}-\frac{1}{4}\left(-\frac{\sqrt{3}}{2}\right)\right) = \frac{\pi}{3}-\frac{\sqrt{3}}{4}$$

15. $\int_0^{\pi/4}\tan^2 x\,dx = \int_0^{\pi/4}(\sec^2 x-1)\,dx = [\tan x - x]_0^{\pi/4} = \left(\tan\left(\frac{\pi}{4}\right)-\frac{\pi}{4}\right)-(\tan(0)-0) = 1-\frac{\pi}{4}$

16. $\int_0^{\pi/6}(\sec x+\tan x)^2\,dx = \int_0^{\pi/6}(\sec^2 x+2\sec x\tan x+\tan^2 x)\,dx = \int_0^{\pi/6}(2\sec^2 x+2\sec x\tan x-1)\,dx$
$= [2\tan x+2\sec x-x]_0^{\pi/6}\left(2\tan\left(\frac{\pi}{6}\right)+2\sec\left(\frac{\pi}{6}\right)-\left(\frac{\pi}{6}\right)\right)-(2\tan 0+2\sec 0-0) = 2\sqrt{3}-\frac{\pi}{6}-2$

17. $\int_0^{\pi/8}\sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/8} = \left(-\frac{1}{2}\cos 2\left(\frac{\pi}{8}\right)\right)-\left(-\frac{1}{2}\cos 2(0)\right) = \frac{2-\sqrt{2}}{4}$

18. $\int_{-\pi/3}^{-\pi/4}\left(4\sec^2 t+\frac{\pi}{t^2}\right)dt = \int_{-\pi/3}^{-\pi/4}(4\sec^2 t+\pi t^{-2})\,dt = \left[4\tan t-\frac{\pi}{t}\right]_{-\pi/3}^{-\pi/4}$
$= \left(4\tan\left(-\frac{\pi}{4}\right)-\frac{\pi}{\left(-\frac{\pi}{4}\right)}\right)-\left(4\tan\left(\frac{\pi}{3}\right)-\frac{\pi}{\left(-\frac{\pi}{3}\right)}\right) = (4(-1)+4)-\left(4\left(-\sqrt{3}\right)+3\right) = 4\sqrt{3}-3$

19. $\int_1^{-1}(r+1)^2\,dr = \int_1^{-1}(r^2+2r+1)\,dr = \left[\frac{r^3}{3}+r^2+r\right]_1^{-1} = \left(\frac{(-1)^3}{3}+(-1)^2+(-1)\right)-\left(\frac{1^3}{3}+1^2+1\right) = -\frac{8}{3}$

20. $\int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)\,dt = \int_{-\sqrt{3}}^{\sqrt{3}}(t^3+t^2+4t+4)\,dt = \left[\frac{t^4}{4}+\frac{t^3}{3}+2t^2+4t\right]_{-\sqrt{3}}^{\sqrt{3}}$
$$= \left(\frac{(\sqrt{3})^4}{4}+\frac{(\sqrt{3})^3}{3}+2(\sqrt{3})^2+4\sqrt{3}\right)-\left(\frac{(-\sqrt{3})^4}{4}+\frac{(-\sqrt{3})^3}{3}+2(-\sqrt{3})^2+4(-\sqrt{3})\right) = 10\sqrt{3}$$

[redacted]

21. $\int_{\sqrt{2}}^{1}\left(\frac{u^7}{2}-\frac{1}{u^5}\right)du = \int_{\sqrt{2}}^{1}\left(\frac{u^7}{2}-u^{-5}\right)du = \left[\frac{u^8}{16}+\frac{1}{4u^4}\right]_{\sqrt{2}}^{1} = \left(\frac{1^8}{16}+\frac{1}{4(1)^4}\right)-\left(\frac{(\sqrt{2})^8}{16}+\frac{1}{4\left(\sqrt{2}\right)^4}\right) = -\frac{3}{4}$

22. $\int_{-3}^{-1}\frac{y^5-2y}{y^3}\,dy = \int_{-3}^{-1}(y^2-2y^{-2})\,dy = \left[\frac{y^3}{3}+2y^{-1}\right]_{-3}^{-1} = \left(\frac{(-1)^3}{3}+\frac{2}{(-1)}\right)-\left(\frac{(-3)^3}{3}+\frac{2}{(-3)}\right)=\frac{22}{3}$

23. $\int_{1}^{\sqrt{2}}\frac{s^2+\sqrt{s}}{s^2}\,ds = \int_{1}^{\sqrt{2}}(1+s^{-3/2})\,ds = \left[s-\frac{2}{\sqrt{s}}\right]_{1}^{\sqrt{2}} = \left(\sqrt{2}-\frac{2}{\sqrt{\sqrt{2}}}\right)-\left(1-\frac{2}{\sqrt{1}}\right)=\sqrt{2}-2^{3/4}+1=\sqrt{2}-\sqrt[4]{8}+1$

24. $\int_{1}^{8}\frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}\,dx = \int_{1}^{8}\frac{2x^{1/3}-x+2-x^{2/3}}{x^{1/3}}\,dx = \int_{1}^{8}(2-x^{2/3}+2x^{-1/3}-x^{1/3})\,dx = \left[2x-\frac{3}{5}x^{5/3}+3x^{2/3}-\frac{3}{4}x^{4/3}\right]_{1}^{3}$
$=\left(2(8)-\frac{3}{5}(8)^{5/3}+3(8)^{2/3}-\frac{3}{4}(8)^{4/3}\right)-\left(2(1)-\frac{3}{5}(1)^{5/3}+3(1)^{2/3}-\frac{3}{4}(1)^{4/3}\right)=-\frac{137}{20}$

25. $\int_{\pi/2}^{\pi}\frac{\sin 2x}{2\sin x}\,dx = \int_{\pi/2}^{\pi}\frac{2\sin x\cos x}{2\sin x}\,dx = \int_{\pi/2}^{\pi}\cos x\,dx = \left[\sin x\right]_{\pi/2}^{\pi} = (\sin(\pi))-\left(\sin\left(\frac{\pi}{2}\right)\right)=-1$

26. $\int_{0}^{\pi/3}(\cos x+\sec x)^2\,dx = \int_{0}^{\pi/3}(\cos^2 x+2+\sec^2 x)\,dx = \int_{0}^{\pi/3}\left(\frac{\cos 2x+1}{2}+2+\sec^2 x\right)dx$
$=\int_{0}^{\pi/3}\left(\frac{1}{2}\cos 2x+\frac{5}{2}+\sec^2 x\right)dx = \left[\frac{1}{4}\sin 2x+\frac{5}{2}x+\tan x\right]_{0}^{\pi/3}$
$=\left(\frac{1}{4}\sin 2\left(\frac{\pi}{3}\right)+\frac{5}{2}\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{3}\right)\right)-\left(\frac{1}{4}\sin 2(0)+\frac{5}{2}(0)+\tan(0)\right)=\frac{5\pi}{6}+\frac{9\sqrt{3}}{8}$

27. $\int_{-4}^{4}|x|\,dx = \int_{-4}^{0}|x|\,dx+\int_{0}^{4}|x|\,dx = -\int_{-4}^{0}x\,dx+\int_{0}^{4}x\,dx = \left[-\frac{x^2}{2}\right]_{-4}^{0}+\left[\frac{x^2}{2}\right]_{0}^{4} = \left(-\frac{0^2}{2}+\frac{(-4)^2}{2}\right)+\left(\frac{4^2}{2}-\frac{0^2}{2}\right)=16$

28. $\int_{0}^{\pi}\frac{1}{2}\left(\cos x+|\cos x|\right)dx = \int_{0}^{\pi/2}\frac{1}{2}(\cos x+\cos x)\,dx+\int_{\pi/2}^{\pi}\frac{1}{2}(\cos x-\cos x)\,dx = \int_{0}^{\pi/2}\cos x\,dx = [\sin x]_{0}^{\pi/2}$
$=\sin\frac{\pi}{2}-\sin 0=1$

29. $\int_{0}^{\ln 2}e^{3x}dx = \frac{1}{3}e^{3x}\Big|_{0}^{\ln 2} = \frac{1}{3}e^{3\ln 2}-\frac{1}{3}e^{0} = \frac{1}{3}e^{\ln 8}-\frac{1}{3}=\frac{8}{3}-\frac{1}{3}=\frac{7}{3}$

30. $\int_{1}^{2}\left(\frac{1}{x}-e^{-x}\right)dx = (\ln x+e^{-x})\Big|_{1}^{2} = (\ln 2+e^{-2})-(\ln 1+e^{-1}) = \ln 2+\frac{1}{e^2}-\frac{1}{e}$

31. $\int_{0}^{1/2}\frac{4}{\sqrt{1-x^2}}dx = 4\sin^{-1}x\Big|_{0}^{1/2} = 4\sin^{-1}\left(\frac{1}{2}\right)-4\sin^{-1}0 = 4\left(\frac{\pi}{6}\right)-4(0)=\frac{2\pi}{3}$

32. $\int_{0}^{1/\sqrt{3}}\frac{dx}{1+4x^2} = \int_{0}^{1/\sqrt{3}}\frac{dx}{1+2^2x^2} = \frac{1}{2}\tan^{-1}(2x)\Big|_{0}^{1/\sqrt{3}} = \frac{1}{2}\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)-\frac{1}{2}\tan^{-1}(0) = \frac{1}{2}\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

33. $\int_{2}^{4}x^{\pi-1}dx = \frac{x^{\pi}}{\pi}\Big|_{2}^{4} = \frac{1}{\pi}(4^{\pi}-2^{\pi})$

[redacted]

34. $\int_{-1}^{0} \pi^{x-1}\,dx = \frac{1}{\ln \pi} \cdot \pi^{x-1}\Big|_{-1}^{0} = \frac{1}{\ln \pi}(\pi^{-1} - \pi^{-2})$

35. $\int_0^1 x e^{x^2}\,dx = \frac{1}{2}e^{x^2}\Big|_0^1 = \frac{1}{2}e^1 - \frac{1}{2}e^0 = \frac{1}{2}(e-1)$

36. $\int_1^2 \frac{\ln x}{x}\,dx = \frac{1}{2}(\ln x)^2\Big|_1^2 = \frac{1}{2}(\ln 2)^2 - \frac{1}{2}(\ln 1)^2 = \frac{1}{2}(\ln 2)^2$

37. $\int_2^5 \frac{x}{\sqrt{1+x^2}}\,dx = \int_2^5 x(1+x^2)^{1/2}\,dx = \sqrt{1+x^2}\Big|_2^5 = \sqrt{26} - \sqrt{5}$

38. $\int_0^{\pi/3} \sin^2 x \cos x\,dx = \int_0^{\pi/3} (\sin x)^2 \cos x\,dx = \frac{1}{3}(\sin x)^3\Big|_0^{\pi/3} = \frac{1}{3}\sin^3\left(\frac{\pi}{3}\right) - \frac{1}{3}\sin^3(0) = \frac{\sqrt{3}}{8}$

39. (a) $\int_0^{\sqrt{x}} \cos t\,dt = [\sin t]_0^{\sqrt{x}} = \sin\sqrt{x} - \sin 0 = \sin\sqrt{x} \Rightarrow \frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos t\,dt\right)$
$= \frac{d}{dx}(\sin\sqrt{x}) = \cos\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

(b) $\frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos t\,dt\right) = (\cos\sqrt{x})\left(\frac{d}{dx}(\sqrt{x})\right) = (\cos\sqrt{x})\left(\frac{1}{2}x^{-1/2}\right) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

40. (a) $\int_1^{\sin x} 3t^2\,dt = [t^3]_1^{\sin x} = \sin^3 x - 1 \Rightarrow \frac{d}{dx}\left(\int_1^{\sin x} 3t^2\,dt\right) = \frac{d}{dx}(\sin^3 x - 1) = 3\sin^2 x \cos x$

(b) $\frac{d}{dx}\left(\int_1^{\sin x} 3t^2 dt\right) = (3\sin^2 x)\left(\frac{d}{dx}(\sin x)\right) = 3\sin^2 x \cos x$

41. (a) $\int_0^{t^4} \sqrt{u}\,du = \int_0^{t^4} u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_0^{t^4} = \frac{2}{3}(t^4)^{3/2} - 0 = \frac{2}{3}t^6 \Rightarrow \frac{d}{dt}\left(\int_0^{t^4} \sqrt{u}\,du\right) = \frac{d}{dt}\left(\frac{2}{3}t^6\right) = 4t^5$

(b) $\frac{d}{dt}\left(\int_0^{t^4} \sqrt{u}\,du\right) = \sqrt{t^4}\left(\frac{d}{dt}(t^4)\right) = t^2(4t^3) = 4t^5$

42. (a) $\int_0^{\tan\theta} \sec^2 y\,dy = [\tan y]_0^{\tan\theta} = \tan(\tan\theta) - 0 = \tan(\tan\theta) \Rightarrow \frac{d}{d\theta}\left(\int_0^{\tan\theta} \sec^2 y\,dy\right)$
$= \frac{d}{d\theta}(\tan(\tan\theta)) = (\sec^2(\tan\theta))\sec^2\theta$

(b) $\frac{d}{d\theta}\left(\int_0^{\tan\theta} \sec^2 y\,dy\right) = (\sec^2(\tan\theta))\left(\frac{d}{d\theta}(\tan\theta)\right) = (\sec^2(\tan\theta))\sec^2\theta$

43. (a) $\int_0^{x^3} e^{-t}\,dt = -e^{-t}\Big|_0^{x^3} = -e^{-x^3} + 1 \Rightarrow \frac{d}{dx}\left(\int_0^{x^3} e^{-t}\,dt\right) = \frac{d}{dx}\left(-e^{-x^3} + 1\right) = 3x^2 e^{-x^3}$

(b) $\frac{d}{dx}\left(\int_0^{x^3} e^{-t}\,dt\right) = e^{-x^3} \cdot \frac{d}{dx}(x^3) = 3x^2 e^{-x^3}$

[redacted]

44. (a) $\int_0^{\sqrt{t}}\left(x^4+\frac{3}{\sqrt{1-x^2}}\right)dx = \frac{x^5}{5}+3\sin^{-1}x\Big|_0^{\sqrt{t}} = \frac{1}{5}t^{5/2}+3\sin^{-1}\sqrt{t}$

$\Rightarrow \frac{d}{dt}\int_0^{\sqrt{t}}\left(x^4+\frac{3}{\sqrt{1-x^2}}\right)dx = \frac{d}{dt}\left(\frac{1}{5}t^{5/2}+3\sin^{-1}\sqrt{t}\right) = \frac{1}{5}\cdot\frac{5}{2}t^{3/2}+\frac{3}{\sqrt{1-t}}\cdot\frac{1}{2\sqrt{t}} = \frac{1}{2}t^{3/2}+\frac{3}{2\sqrt{t-t^2}}$

(b) $\frac{d}{dt}\int_0^{\sqrt{t}}\left(x^4+\frac{3}{\sqrt{1-x^2}}\right)dx = \left(t^2+\frac{3}{\sqrt{1-t}}\right)\cdot\frac{d}{dt}\left(\sqrt{t}\right) = \left(t^2+\frac{3}{\sqrt{1-t}}\right)\cdot\frac{1}{2\sqrt{t}} = \frac{1}{2}t^{3/2}+\frac{3}{2\sqrt{t-t^2}}$

45. $y=\int_0^x\sqrt{1+t^2}\,dt \Rightarrow \frac{dy}{dx}=\sqrt{1+x^2}$

46. $y=\int_1^x\frac{1}{t}\,dt \Rightarrow \frac{dy}{dx}=\frac{1}{x},\ x>0$

47. $y=\int_{\sqrt{x}}^0\sin t^2dt = -\int_0^{\sqrt{x}}\sin t^2\,dt \Rightarrow \frac{dy}{dx} = -\left(\sin(\sqrt{x})^2\right)\left(\frac{d}{dx}(\sqrt{x})\right) = -(\sin x)\left(\frac{1}{2}x^{-1/2}\right) = -\frac{\sin x}{2\sqrt{x}}$

48. $y=x\int_2^{x^2}\sin t^3dt \Rightarrow \frac{dy}{dx} = x\cdot\frac{d}{dx}\left(\int_2^{x^2}\sin t^3dt\right)+1\cdot\int_2^{x^2}\sin t^3dt = x\cdot\sin\,(x^2)^3\,\frac{d}{dx}(x^2)+\int_2^{x^2}\sin t^3dt$

$=2x^2\sin x^6+\int_2^{x^2}\sin t^3dt$

49. $y=\int_{-1}^x\frac{t^2}{t^2+4}\,dt-\int_3^x\frac{t^2}{t^2+4}\,dt \Rightarrow \frac{dy}{dx}=\frac{x^2}{x^2+4}-\frac{x^2}{x^2+4}=0$

50. $y=\left(\int_0^x(t^3+1)^{10}dt\right)^3 \Rightarrow \frac{dy}{dx}=3\left(\int_0^x(t^3+1)^{10}dt\right)^2\frac{d}{dx}\left(\int_0^x(t^3+1)^{10}dt\right)=3(x^3+1)^{10}\left(\int_0^x(t^3+1)^{10}dt\right)^2$

51. $y=\int_0^{\sin x}\frac{dt}{\sqrt{1-t^2}},\ |x|<\frac{\pi}{2} \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-\sin^2x}}\left(\frac{d}{dx}(\sin x)\right) = \frac{1}{\sqrt{\cos^2x}}(\cos x)=\frac{\cos x}{|\cos x|}=\frac{\cos x}{\cos x}=1$ since $|x|<\frac{\pi}{2}$

52. $y=\int_0^{\tan x}\frac{dt}{1+t^2} \Rightarrow \frac{dy}{dx}=\left(\frac{1}{1+\tan^2x}\right)\left(\frac{d}{dx}(\tan x)\right)=\left(\frac{1}{\sec^2x}\right)\left(\sec^2x\right)=1$

53. $y=\int_0^{e^{x^2}}\frac{1}{\sqrt{t}}\,dt \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{e^{x^2}}}\cdot\frac{d}{dx}\left(e^{x^2}\right)=\frac{1}{e^{\frac{1}{2}x^2}}\cdot 2xe^{x^2} = 2xe^{\frac{1}{2}x^2}$

54. $y=\int_{2^x}^1\sqrt[3]{t}\,dt=-\int_1^{2^x}t^{1/3}dt \Rightarrow \frac{dy}{dx}=-(2^x)^{1/3}\cdot\frac{d}{dx}(2^x)=-2^{x/3}\cdot 2^x\ln 2=-2^{4x/3}\ln 2$

55. $y=\int_0^{\sin^{-1}t}\cos t\,dt \Rightarrow \frac{dy}{dx}=\cos(\sin^{-1}x)\cdot\frac{d}{dx}(\sin^{-1}x)=\sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}}=1$

56. $y=\int_{-1}^{x^{1/\pi}}\sin^{-1}t\,dt \Rightarrow \frac{dy}{dx}=\sin^{-1}\left(x^{\frac{1}{\pi}}\right)\cdot\frac{d}{dx}\left(x^{\frac{1}{\pi}}\right)=\sin^{-1}\left(x^{\frac{1}{\pi}}\right)\cdot\frac{1}{\pi}x^{\frac{1}{\pi}-1}$

[redacted]

57. $-x^2-2x=0 \Rightarrow -x(x+2)=0 \Rightarrow x=0$ or $x=-2$;

Area $= -\int_{-3}^{-2}(-x^2-2x)dx+\int_{-2}^{0}(-x^2-2x)dx$

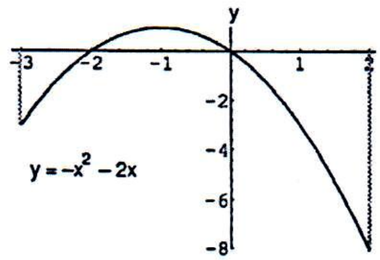
$-\int_{0}^{2}(-x^2-2x)dx$

$= -\left[-\frac{x^3}{3}-x^2\right]_{-3}^{-2}+\left[-\frac{x^3}{3}-x^2\right]_{-2}^{0}-\left[-\frac{x^3}{3}-x^2\right]_{0}^{2}$

$= -\left(\left(-\frac{(-2)^3}{3}-(-2)^2\right)-\left(-\frac{(-3)^3}{3}-(-3)^2\right)\right)$

$+\left(\left(-\frac{0^3}{3}-0^2\right)-\left(-\frac{(-2)^3}{3}-(-2)^2\right)\right)$

$-\left(\left(-\frac{2^3}{3}-2^2\right)-\left(-\frac{0^3}{3}-0^2\right)\right)=\frac{28}{3}$



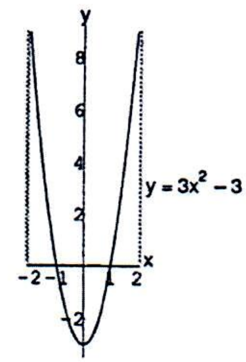
58. $3x^2-3=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$;
because of symmetry about the y-axis,

Area $= 2\left(-\int_0^1(3x^2-3)dx+\int_1^2(3x^2-3)dx\right)$

$2\left(-[x^3-3x]_0^1+[x^3-3x]_1^2\right)$

$= 2[-((1^3-3(1))-(0^3-3(0)))+((2^3-3(2))-(1^3-3(1))]$

$= 2(6) = 12$

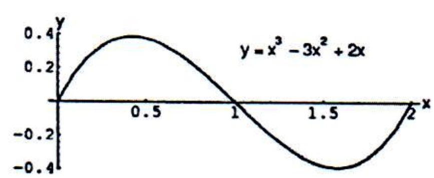


59. $x^3-3x^2+2x=0 \Rightarrow x(x^2-3x+2)=0$
$\Rightarrow x(x-2)(x-1)=0 \Rightarrow x=0, 1,$ or 2;

Area $= \int_0^1(x^3-3x^2+2x)dx-\int_1^2(x^3-3x^2+2x)dx$

$= \left[\frac{x^4}{4}-x^3+x^2\right]_0^1-\left[\frac{x^4}{4}-x^3+x^2\right]_1^2=\left(\frac{1^4}{4}-1^3+1^2\right)-\left(\frac{0^4}{4}-0^3+0^2\right)$

$-\left[\left(\frac{2^4}{4}-2^3+2^2\right)-\left(\frac{1^4}{4}-1^3+1^2\right)\right]=\frac{1}{2}$



60. $x^{1/3}-x=0 \Rightarrow x^{1/3}\left(1-x^{2/3}\right)=0 \Rightarrow x^{1/3}=0$ or $1-x^{2/3}=0 \Rightarrow x=0$
or $1=x^{2/3} \Rightarrow x=0$ or $1=x^2 \Rightarrow x=0$ or $x=\pm 1$;

Area $= -\int_{-1}^{0}(x^{1/3}-x)\,dx+\int_0^1(x^{1/3}-x)\,dx-\int_1^8(x^{1/3}-x)\,dx$

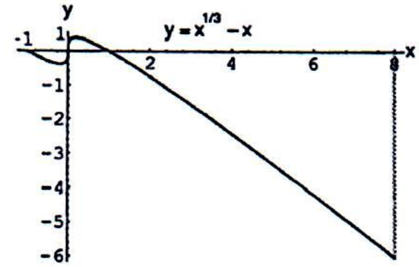
$= -\left[\frac{3}{4}x^{4/3}-\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{3}{4}x^{4/3}-\frac{x^2}{2}\right]_0^1-\left[\frac{3}{4}x^{4/3}-\frac{x^2}{2}\right]_1^8$

$= -\left[\left(\frac{3}{4}(0)^{4/3}-\frac{0^2}{2}\right)-\left(\frac{3}{4}(-1)^{4/3}-\frac{(-1)^2}{2}\right)\right]$

$+\left[\left(\frac{3}{4}(1)^{4/3}-\frac{1^2}{2}\right)-\left(\frac{3}{4}(0)^{4/3}-\frac{0^2}{2}\right)\right]$

$-\left[\left(\frac{3}{4}(8)^{4/3}-\frac{8^2}{2}\right)-\left(\frac{3}{4}(1)^{4/3}-\frac{1^2}{2}\right)\right]$

$= \frac{1}{4}+\frac{1}{4}-(-20-\frac{3}{4}+\frac{1}{2})=\frac{83}{4}$



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61. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $x = \pi$, and $x = 0$ is 2π. The area under the curve $y = 1 + \cos x$ on $[0, \pi]$ is $\int_0^{\pi} (1 + \cos x)\, dx = [x + \sin x]_0^{\pi} = (\pi + \sin \pi) - (0 + \sin 0) = \pi$. Therefore the area of the shaded region is $2\pi - \pi = \pi$.

62. The area of the rectangle bounded by the lines by the lines $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$, $y = \sin\frac{\pi}{6} = \frac{1}{2} = \sin\frac{5\pi}{6}$, and $y = 0$ is $\frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{\pi}{3}$. The area under the curve $y = \sin x$ on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is $\int_{\pi/6}^{5\pi/6} \sin x\, dx = [-\cos x]_{\pi/6}^{5\pi/6}$ $= \left(-\cos\frac{5\pi}{6}\right) - \left(-\cos\frac{\pi}{6}\right) = -\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} = \sqrt{3}$. Therefore the area of the shaded region is $\sqrt{3} - \frac{\pi}{3}$.

63. On $\left[-\frac{\pi}{4}, 0\right]$: The area of the rectangle bounded by the lines $y = \sqrt{2}$, $y = 0$, $\theta = 0$, and $\theta = -\frac{\pi}{4}$ is $\sqrt{2}\left(\frac{\pi}{4}\right)$ $= \frac{\pi\sqrt{2}}{4}$. The area between the curve $y = \sec\theta \tan\theta$ and $y = 0$ is $-\int_{-\pi/4}^{0} \sec\theta \tan\theta\, d\theta = [-\sec\theta]_{-\pi/4}^{0}$ $= (-\sec 0) - \left(-\sec\left(-\frac{\pi}{4}\right)\right) = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[-\frac{\pi}{4}, 0\right]$ is $\frac{\pi\sqrt{2}}{4} + (\sqrt{2} - 1)$. On $\left[0, \frac{\pi}{4}\right]$: The area of the rectangle bounded by $\theta = \frac{\pi}{4}$, $\theta = 0$, $y = \sqrt{2}$, and $y = 0$ is $\sqrt{2}\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{4}$. The area under the curve $y = \sec\theta \tan\theta$ is $\int_0^{\pi/4} \sec\theta \tan\theta\, d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[0, \frac{\pi}{4}\right]$ is $\frac{\pi\sqrt{2}}{4} - (\sqrt{2} - 1)$. Thus, the area of the total shaded region is $\left(\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1\right) + \left(\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1\right) = \frac{\pi\sqrt{2}}{2}$.

64. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $t = -\frac{\pi}{4}$, and $t = 1$ is $2\left(1 - \left(-\frac{\pi}{4}\right)\right) = 2 + \frac{\pi}{2}$. The area under the curve $y = \sec^2 t$ on $\left[-\frac{\pi}{4}, 0\right]$ is $\int_{-\pi/4}^{0} \sec^2 t\, dt = [\tan t]_{-\pi/4}^{0} = \tan 0 - \tan\left(-\frac{\pi}{4}\right) = 1$. The area under the curve $y = 1 - t^2$ on $[0, 1]$ is $\int_0^1 (1 - t^2)\, dt = \left[t - \frac{t^2}{3}\right]_0^1 = \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) = \frac{2}{3}$. Thus, the total area under the curves on $\left[-\frac{\pi}{4}, 1\right]$ is $1 + \frac{2}{3} = \frac{5}{3}$. Therefore the area of the shaded region is $\left(2 + \frac{\pi}{2}\right) - \frac{5}{3} = \frac{1}{3} + \frac{\pi}{2}$.

65. $y = \int_{\pi}^{x} \frac{1}{t}\, dt - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ and $y(\pi) = \int_{\pi}^{\pi} \frac{1}{t} dt - 3 = 0 - 3 = -3 \Rightarrow$ (*d*) is a solution to this problem.

66. $y = \int_{-1}^{x} \sec t\, dt + 4 \Rightarrow \frac{dy}{dx} = \sec x$ and $y(-1) = \int_{-1}^{-1} \sec t\, dt + 4 = 0 + 4 = 4 \Rightarrow$ (*c*) is a solution to this problem.

67. $y = \int_0^x \sec t\, dt + 4 \Rightarrow \frac{dy}{dx} = \sec x$ and $y(0) = \int_0^0 \sec t\, dt + 4 = 0 + 4 = 4 \Rightarrow$ (*b*) is a solution to this problem.

68. $y = \int_1^x \frac{1}{t}\, dt - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ and $y(1) = \int_1^1 \frac{1}{t}\, dt - 3 = 0 - 3 = -3 \Rightarrow$ (*a*) is a solution to this problem.

69. $y = \int_2^x \sec t\, dt + 3$

70. $y = \int_1^x \sqrt{1 + t^2}\, dt - 2$

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