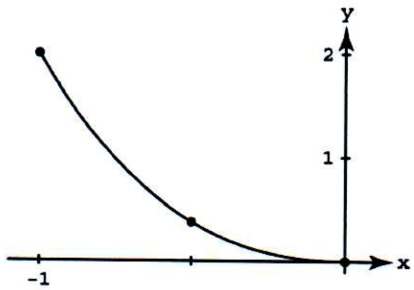
46. $f(x) = x^2 - x^3$



Let $\Delta x = \frac{0-(-1)}{n} = \frac{1}{n}$ and $c_i = -1 + i\Delta x = -1 + \frac{i}{n}$. The right-hand sum is $\sum_{i=1}^{n} (c_i^2 - c_i^3)\frac{1}{n} = \sum_{i=1}^{n} \left(\left(-1 + \frac{i}{n}\right)^2 - \left(-1 + \frac{i}{n}\right)^3\right)\frac{1}{n}$

$= \sum_{i=1}^{n} \left(2 - \frac{5i}{n} + \frac{4i^2}{n^2} - \frac{i^3}{n^3}\right)\frac{1}{n} = \sum_{i=1}^{n} \left(\frac{2}{n} - \frac{5i}{n^2} + \frac{4i^2}{n^3} - \frac{i^3}{n^4}\right)$

$= \sum_{i=1}^{n} \frac{2}{n} - \frac{5}{n^2}\sum_{i=1}^{n} i + \frac{4}{n^3}\sum_{i=1}^{n} i^2 - \frac{1}{n^4}\sum_{i=1}^{n} i^3$

$= \frac{2}{n}(n) - \frac{5}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{4}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2$

$= 2 - \frac{5n+5}{2n} + \frac{4n^2+6n+2}{3n^2} - \frac{n^2+2n+1}{4n^2} = 2 - \frac{5+\frac{5}{n}}{2} + \frac{4+\frac{6}{n}+\frac{2}{n^2}}{3} - \frac{1+\frac{2}{n}+\frac{1}{n^2}}{4}$. Thus, $\lim_{n\to\infty} \sum_{i=1}^{n} (c_i^2 - c_i^3)\frac{1}{n}$

$= \lim_{n\to\infty} \left[2 - \frac{5+\frac{5}{n}}{2} + \frac{4+\frac{6}{n}+\frac{2}{n^2}}{3} - \frac{1+\frac{2}{n}+\frac{1}{n^2}}{4}\right] = 2 - \frac{5}{2} + \frac{4}{3} - \frac{1}{4} = \frac{7}{12}$.

5.3 THE DEFINITE INTEGRAL

1. $\int_0^2 x^2\,dx$

2. $\int_{-1}^{0} 2x^3\,dx$

3. $\int_{-7}^{5} (x^2 - 3x)\,dx$

4. $\int_1^4 \frac{1}{x}\,dx$

5. $\int_2^3 \frac{1}{1-x}\,dx$

6. $\int_0^1 \sqrt{4 - x^2}\,dx$

7. $\int_{-\pi/4}^{0} (\sec x)\,dx$

8. $\int_0^{\pi/4} (\tan x)\,dx$

9. (a) $\int_2^2 g(x)\,dx = 0$

(b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$

(c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = 3(-4) = -12$

(d) $\int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx = 6 - (-4) = 10$

(e) $\int_1^5 [f(x) - g(x)]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 6 - 8 = -2$

(f) $\int_1^5 [4f(x) - g(x)]\,dx = 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 4(6) - 8 = 16$

10. (a) $\int_1^9 -2f(x)\,dx = -2\int_1^9 f(x)\,dx = -2(-1) = 2$

(b) $\int_7^9 [f(x) + h(x)]\,dx = \int_7^9 f(x)\,dx + \int_7^9 h(x)\,dx = 5 + 4 = 9$

(c) $\int_7^9 [2f(x) - 3h(x)]\,dx = 2\int_7^9 f(x)\,dx - 3\int_7^9 h(x)\,dx = 2(5) - 3(4) = -2$

(d) $\int_9^1 f(x)\,dx = -\int_1^9 f(x)\,dx = -(-1) = 1$

(e) $\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx = -1 - 5 = -6$

(f) $\int_9^7 [h(x) - f(x)]\,dx = \int_7^9 [f(x) - h(x)]\,dx = \int_7^9 f(x)\,dx - \int_7^9 h(x)\,dx = 5 - 4 = 1$

11. (a) $\int_1^2 f(u)\,du = \int_1^2 f(x)\,dx = 5$

(b) $\int_1^2 \sqrt{3}\,f(z)\,dz = \sqrt{3}\int_1^2 f(z)\,dz = 5\sqrt{3}$

(c) $\int_2^1 f(t)\,dt = -\int_1^2 f(t)\,dt = -5$

(d) $\int_1^2 [-f(x)]\,dx = -\int_1^2 f(x)\,dx = -5$

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12. (a) $\int_0^{-3} g(t)\,dt = -\int_{-3}^{0} g(t)\,dt = -\sqrt{2}$

(b) $\int_{-3}^{0} g(u)\,du = \int_{-3}^{0} g(t)\,dt = \sqrt{2}$

(c) $\int_{-3}^{0} [-g(x)]\,dx = -\int_{-3}^{0} g(x)\,dx = -\sqrt{2}$

(d) $\int_{-3}^{0} \frac{g(r)}{\sqrt{2}}\,dr = \frac{1}{\sqrt{2}} \int_{-3}^{0} g(t)\,dt = \left(\frac{1}{\sqrt{2}}\right)\left(\sqrt{2}\right) = 1$

13. (a) $\int_3^4 f(z)\,dz = \int_0^4 f(z)\,dz - \int_0^3 f(z)\,dz = 7 - 3 = 4$

(b) $\int_4^3 f(t)\,dt = -\int_3^4 f(t)\,dt = -4$

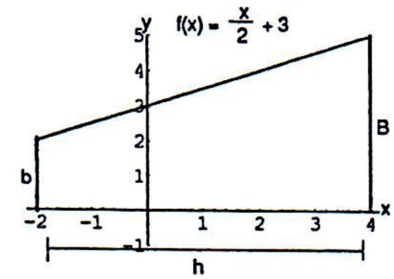
14. (a) $\int_1^3 h(r)\,dr = \int_{-1}^{3} h(r)\,dr - \int_{-1}^{1} h(r)\,dr = 6 - 0 = 6$

(b) $-\int_3^1 h(u)\,du = -\left(-\int_1^3 h(u)\,du\right) = \int_1^3 h(u)\,du = 6$

15. The area of the trapezoid is $A = \frac{1}{2}(B + b)h$

$= \frac{1}{2}(5 + 2)(6) = 21 \Rightarrow \int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

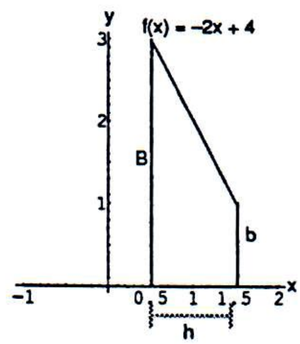
$= 21$ square units



16. The area of the trapezoid is $A = \frac{1}{2}(B + b)h$

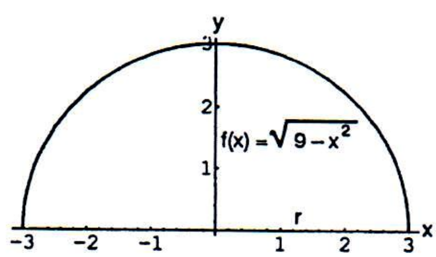
$= \frac{1}{2}(3 + 1)(1) = 2 \Rightarrow \int_{1/2}^{3/2} (-2x + 4)\,dx$

$= 2$ square units



17. The area of the semicircle is $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(3)^2$

$= \frac{9}{2}\pi \Rightarrow \int_{-3}^{3} \sqrt{9 - x^2}\,dx = \frac{9}{2}\pi$ square units



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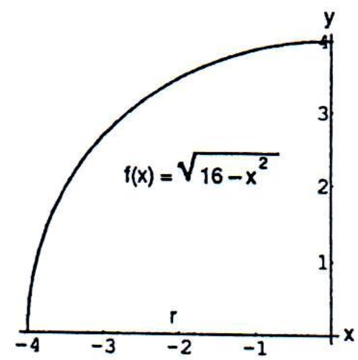
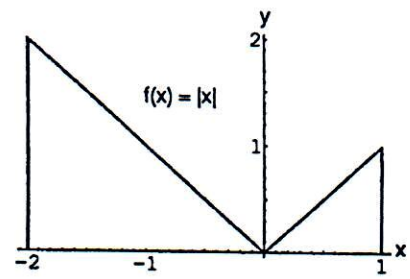
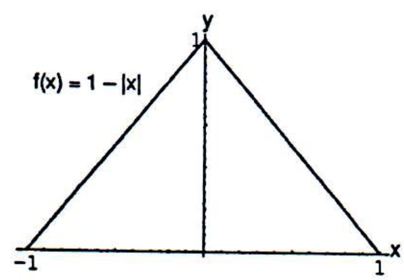
18. The graph of the quarter circle is $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(4)^2$
$= 4\pi \Rightarrow \int_{-4}^{0} \sqrt{16 - x^2}\,dx = 4\pi$ square units



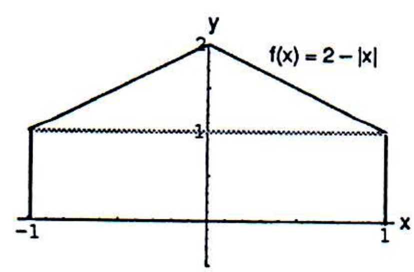
19. The area of the triangle on the left is $A = \frac{1}{2}bh = \frac{1}{2}(2)(2)$
$= 2$. The area of the triangle on the right is $A = \frac{1}{2}bh$
$= \frac{1}{2}(1)(1) = \frac{1}{2}$. Then, the total area is 2.5
$\Rightarrow \int_{-2}^{1} |x|\,dx = 2.5$ square units



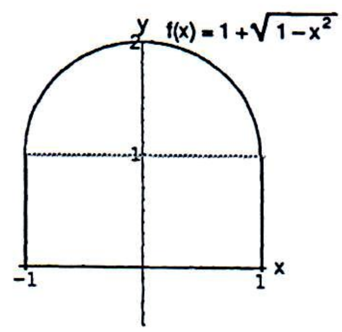
20. The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$
$\Rightarrow \int_{-1}^{1} (1 - |x|)\,dx = 1$ square unit



21. The area of the triangular peak is $A = \frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$.
The area of the rectangular base is $S = \ell w = (2)(1) = 2$.
Then the total area is 3 $\Rightarrow \int_{-1}^{1} (2 - |x|)\,dx = 3$ square units



22. $y = 1 + \sqrt{1 - x^2} \Rightarrow y - 1 = \sqrt{1 - x^2}$
$\Rightarrow (y - 1)^2 = 1 - x^2 \Rightarrow x^2 + (y - 1)^2 = 1$, a circle with center (0, 1) and radius of 1 $\Rightarrow y = 1 + \sqrt{1 - x^2}$ is the upper semicircle. The area of this semicircle is
$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$. The area of the rectangular base is $A = \ell w = (2)(1) = 2$. Then the total area is $2 + \frac{\pi}{2}$
$\Rightarrow \int_{-1}^{1} \left(1 + \sqrt{1 - x^2}\right) dx = 2 + \frac{\pi}{2}$ square units



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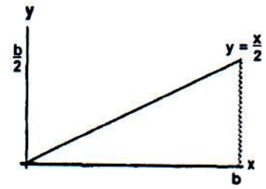
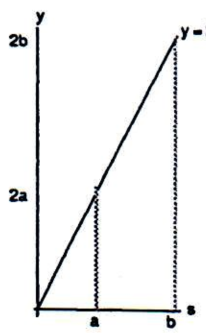
23. $\int_0^b \frac{x}{2}\,dx = \frac{1}{2}(b)\left(\frac{b}{2}\right) = \frac{b^2}{4}$

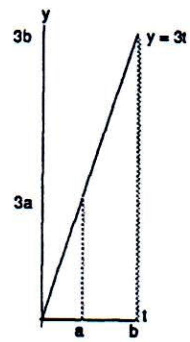


24. $\int_0^b 4x\,dx = \frac{1}{2}b(4b) = 2b^2$

25. $\int_a^b 2s\,ds = \frac{1}{2}b(2b) - \frac{1}{2}a(2a) = b^2 - a^2$



26. $\int_a^b 3t\,dt = \frac{1}{2}b(3b) - \frac{1}{2}a(3a) = \frac{3}{2}(b^2 - a^2)$



27. (a) $\int_{-2}^{2} \sqrt{4 - x^2}\,dx = \frac{1}{2}\left[\pi(2)^2\right] = 2\pi$ (b) $\int_0^2 \sqrt{4 - x^2}\,dx = \frac{1}{4}\left[\pi(2)^2\right] = \pi$

28. (a) $\int_{-1}^{0}\left(3x + \sqrt{1 - x^2}\right)dx = \int_{-1}^{0} 3x\,dx + \int_{-1}^{0}\sqrt{1 - x^2}\,dx = -\frac{1}{2}\left[(1)(3)\right] + \frac{1}{4}\left[\pi(1)^2\right] = \frac{\pi}{4} - \frac{3}{2}$

(b) $\int_{-1}^{0}\left(3x + \sqrt{1 - x^2}\right)dx = \int_{-1}^{0} 3x\,dx + \int_0^1 3x\,dx + \int_{-1}^{1}\sqrt{1 - x^2}\,dx = -\frac{1}{2}\left[(1)(3)\right] + \frac{1}{2}\left[(1)(3)\right] + \frac{1}{2}\left[\pi(1)^2\right] = \frac{\pi}{2}$

29. $\int_1^{\sqrt{2}} x\,dx = \frac{\left(\sqrt{2}\right)^2}{2} - \frac{(1)^2}{2} = \frac{1}{2}$

30. $\int_{0.5}^{2.5} x\,dx = \frac{(2.5)^2}{2} - \frac{(0.5)^2}{2} = 3$

31. $\int_\pi^{2\pi} \theta\,d\theta = \frac{(2\pi)^2}{2} - \frac{\pi^2}{2} = \frac{3\pi^2}{2}$

32. $\int_{\sqrt{2}}^{5\sqrt{2}} r\,dr = \frac{\left(5\sqrt{2}\right)^2}{2} - \frac{\left(\sqrt{2}\right)^2}{2} = 24$

33. $\int_0^{\sqrt[3]{7}} x^2\,dx = \frac{\left(\sqrt[3]{7}\right)^3}{3} = \frac{7}{3}$

34. $\int_0^{0.3} s^2\,ds = \frac{(0.3)^3}{3} = 0.009$

35. $\int_0^{1/2} t^2\,dt = \frac{\left(\frac{1}{2}\right)^3}{3} = \frac{1}{24}$

36. $\int_0^{\pi/2} \theta^2\,d\theta = \frac{\left(\frac{\pi}{2}\right)^3}{3} = \frac{\pi^3}{24}$

37. $\int_a^{2a} x\,dx = \frac{(2a)^2}{2} - \frac{a^2}{2} = \frac{3a^2}{2}$

38. $\int_a^{\sqrt{3}a} x\,dx = \frac{\left(\sqrt{3}a\right)^2}{2} - \frac{a^2}{2} = a^2$

39. $\int_0^{\sqrt[3]{b}} x^2\,dx = \frac{\left(\sqrt[3]{b}\right)^3}{3} = \frac{b}{3}$

40. $\int_0^{3b} x^2\,dx = \frac{(3b)^3}{3} = 9b^3$

41. $\int_3^1 7\,dx = 7(1-3) = -14$

42. $\int_0^2 5x\,dx = 5\int_0^2 x\,dx = 5\left[\frac{2^2}{2}-\frac{0^2}{2}\right] = 10$

43. $\int_0^2 (2t-3)\,dt = 2\int_1^1 t\,dt - \int_0^2 3\,dt = 2\left[\frac{2^2}{2}-\frac{0^2}{2}\right] - 3(2-0) = 4-6 = -2$

44. $\int_0^{\sqrt{2}} \left(t-\sqrt{2}\right) dt = \int_0^{\sqrt{2}} t\,dt - \int_0^{\sqrt{2}} \sqrt{2}\,dt = \left[\frac{\left(\sqrt{2}\right)^2}{2} - \frac{0^2}{2}\right] - \sqrt{2}\left[\sqrt{2}-0\right] = 1-2 = -1$

45. $\int_2^1 \left(1+\frac{z}{2}\right) dz = \int_2^1 1\,dz + \int_2^1 \frac{z}{2}\,dz = \int_2^1 1\,dz - \frac{1}{2}\int_1^2 z\,dz = 1[1-2] - \frac{1}{2}\left[\frac{2^2}{2}-\frac{1^2}{2}\right] = -1-\frac{1}{2}\left(\frac{3}{2}\right) = -\frac{7}{4}$

46. $\int_3^0 (2z-3)\,dz = \int_3^0 2z\,dz - \int_3^0 3\,dz = -2\int_0^3 z\,dz - \int_3^0 3\,dz = -2\left[\frac{3^2}{2}-\frac{0^2}{2}\right] - 3[0-3] = -9+9 = 0$

47. $\int_1^2 3u^2\,du = 3\int_1^2 u^2\,du = 3\left[\int_0^2 u^2\,du - \int_0^1 u^2\,du\right] = 3\left(\left[\frac{2^3}{3}-\frac{0^3}{3}\right] - \left[\frac{1^3}{3}-\frac{0^3}{3}\right]\right) = 3\left[\frac{2^3}{3}-\frac{1^3}{3}\right] = 3\left(\frac{7}{3}\right) = 7$

48. $\int_{1/2}^1 24u^2\,du = 24\int_{1/2}^1 u^2\,du = 24\left[\int_0^1 u^2\,du - \int_0^{1/2} u^2\,du\right] = 24\left[\frac{1^3}{3} - \frac{\left(\frac{1}{2}\right)^3}{3}\right] = 24\left[\frac{\left(\frac{7}{8}\right)}{3}\right] = 7$

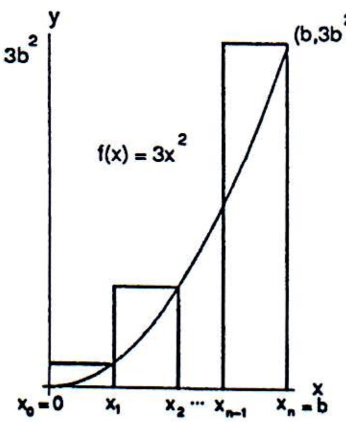
49. $\int_0^2 (3x^2+x-5)\,dx = 3\int_0^2 x^2\,dx + \int_0^2 x\,dx - \int_0^2 5\,dx = 3\left[\frac{2^3}{3}-\frac{0^3}{3}\right] + \left[\frac{2^2}{2}-\frac{0^2}{2}\right] - 5[2-0] = (8+2)-10 = 0$

50. $\int_1^0 (3x^2+x-5)\,dx = -\int_0^1 (3x^2+x-5)\,dx = -\left[3\int_0^1 x^2\,dx + \int_0^1 x\,dx - \int_0^1 5\,dx\right]$
$= -\left[3\left(\frac{1^3}{3}-\frac{0^3}{3}\right) + \left(\frac{1^2}{2}-\frac{0^2}{2}\right) - 5(1-0)\right] = -\left(\frac{3}{2}-5\right) = \frac{7}{2}$

51. Let $\Delta x = \frac{b-0}{n} = \frac{b}{n}$ and let $x_0 = 0$, $x_1 = \Delta x$, $x_2 = 2\Delta x, \ldots, x_{n-1} = (n-1)\Delta x$, $x_n = n\Delta x = b$. Let the c_k's be the right end-points of the subintervals $\Rightarrow c_1 = x_1$, $c_2 = x_2$, and so on. The rectangles defined have areas:

$f(c_1)\,\Delta x = f(\Delta x)\,\Delta x = 3(\Delta x)^2\,\Delta x = 3(\Delta x)^3$
$f(c_2)\,\Delta x = f(2\Delta x)\,\Delta x = 3(2\Delta x)^2\,\Delta x = 3(2)^2(\Delta x)^3$
$f(c_3)\,\Delta x = f(3\Delta x)\,\Delta x = 3(3\Delta x)^2\,\Delta x = 3(3)^2(\Delta x)^3$
$\vdots$
$f(c_n)\,\Delta x = f(n\Delta x)\,\Delta x = 3(n\Delta x)^2\,\Delta x = 3(n)^2(\Delta x)^3$

Then $S_n = \sum_{k=1}^{n} f(c_k)\,\Delta x = \sum_{k=1}^{n} 3k^2(\Delta x)^3$
$= 3(\Delta x)^3 \sum_{k=1}^{n} k^2 = 3\left(\frac{b^3}{n^3}\right)\left(\frac{n(n+1)(2n+1)}{6}\right)$
$= \frac{b^3}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right) \Rightarrow \int_0^b 3x^2\,dx = \lim_{n\to\infty} \frac{b^3}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right) = b^3.$



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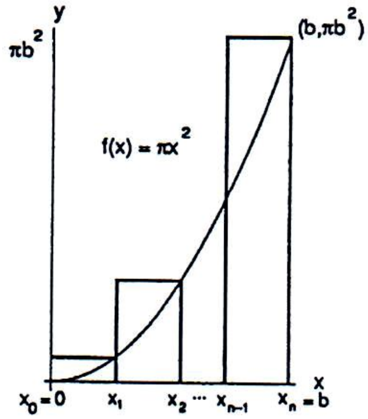
52. Let $\Delta x = \frac{b-0}{n} = \frac{b}{n}$ and let $x_0 = 0$, $x_1 = \Delta x$, $x_2 = 2\Delta x, \ldots, x_{n-1} = (n-1)\Delta x$, $x_n = n\Delta x = b$. Let the c_k's be the right end-points of the subintervals $\Rightarrow c_1 = x_1$, $c_2 = x_2$, and so on. The rectangles defined have areas:

$$f(c_1)\,\Delta x = f(\Delta x)\,\Delta x = \pi(\Delta x)^2\,\Delta x = \pi(\Delta x)^3$$
$$f(c_2)\,\Delta x = f(2\Delta x)\,\Delta x = \pi(2\Delta x)^2\,\Delta x = \pi(2)^2(\Delta x)^3$$
$$f(c_3)\,\Delta x = f(3\Delta x)\,\Delta x = \pi(3\Delta x)^2\,\Delta x = \pi(3)^2(\Delta x)^3$$
$$\vdots$$
$$f(c_n)\,\Delta x = f(n\Delta x)\,\Delta x = \pi(n\Delta x)^2\,\Delta x = \pi(n)^2(\Delta x)^3$$

Then $S_n = \sum_{k=1}^{n} f(c_k)\,\Delta x = \sum_{k=1}^{n} \pi k^2(\Delta x)^3$

$= \pi(\Delta x)^3 \sum_{k=1}^{n} k^2 = \pi\left(\frac{b^3}{n^3}\right)\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \frac{\pi b^3}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) \Rightarrow \int_0^b \pi x^2\,dx = \lim_{n\to\infty} \frac{\pi b^3}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) = \frac{\pi b^3}{3}$.



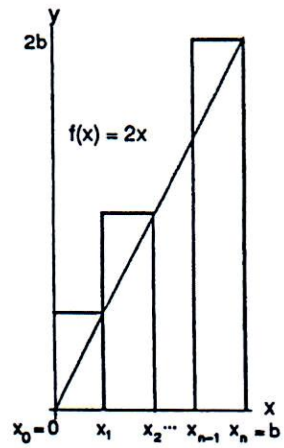
53. Let $\Delta x = \frac{b-0}{n} = \frac{b}{n}$ and let $x_0 = 0$, $x_1 = \Delta x$, $x_2 = 2\Delta x, \ldots, x_{n-1} = (n-1)\Delta x$, $x_n = n\Delta x = b$. Let the c_k's be the right end-points of the subintervals $\Rightarrow c_1 = x_1$, $c_2 = x_2$, and so on. The rectangles defined have areas:

$$f(c_1)\,\Delta x = f(\Delta x)\,\Delta x = 2(\Delta x)(\Delta x) = 2(\Delta x)^2$$
$$f(c_2)\,\Delta x = f(2\Delta x)\,\Delta x = 2(2\Delta x)(\Delta x) = 2(2)(\Delta x)^2$$
$$f(c_3)\,\Delta x = f(3\Delta x)\,\Delta x = 2(3\Delta x)(\Delta x) = 2(3)(\Delta x)^2$$
$$\vdots$$
$$f(c_n)\,\Delta x = f(n\Delta x)\,\Delta x = 2(n\Delta x)(\Delta x) = 2(n)(\Delta x)^2$$

Then $S_n = \sum_{k=1}^{n} f(c_k)\,\Delta x = \sum_{k=1}^{n} 2k(\Delta x)^2$

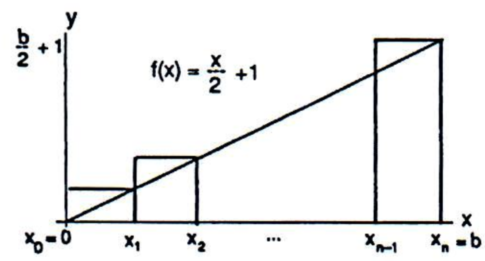
$= 2(\Delta x)^2 \sum_{k=1}^{n} k = 2\left(\frac{b^2}{n^2}\right)\left(\frac{n(n+1)}{2}\right)$

$= b^2\left(1 + \frac{1}{n}\right) \Rightarrow \int_0^b 2x\,dx = \lim_{n\to\infty} b^2\left(1 + \frac{1}{n}\right) = b^2$.



54. Let $\Delta x = \frac{b-0}{n} = \frac{b}{n}$ and let $x_0 = 0$, $x_1 = \Delta x$, $x_2 = 2\Delta x, \ldots, x_{n-1} = (n-1)\Delta x$, $x_n = n\Delta x = b$. Let the c_k's be the right end-points of the subintervals $\Rightarrow c_1 = x_1$, $c_2 = x_2$, and so on. The rectangles defined have areas:

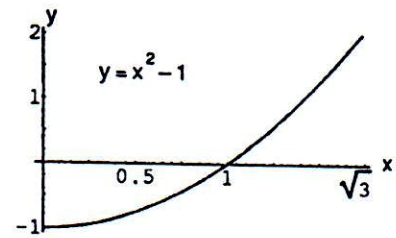
$$f(c_1)\,\Delta x = f(\Delta x)\,\Delta x = \left(\frac{\Delta x}{2} + 1\right)(\Delta x) = \frac{1}{2}(\Delta x)^2 + \Delta x$$
$$f(c_2)\,\Delta x = f(2\Delta x)\,\Delta x = \left(\frac{2\Delta x}{2} + 1\right)(\Delta x) = \frac{1}{2}(2)(\Delta x)^2 + \Delta x$$
$$f(c_3)\,\Delta x = f(3\Delta x)\,\Delta x = \left(\frac{3\Delta x}{2} + 1\right)(\Delta x) = \frac{1}{2}(3)(\Delta x)^2 + \Delta x$$
$$\vdots$$
$$f(c_n)\,\Delta x = f(n\Delta x)\,\Delta x = \left(\frac{n\Delta x}{2} + 1\right)(\Delta x) = \frac{1}{2}(n)(\Delta x)^2 + \Delta x$$



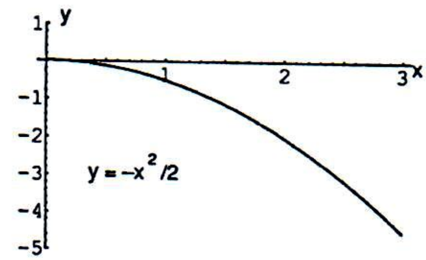
Then $S_n = \sum_{k=1}^{n} f(c_k)\,\Delta x = \sum_{k=1}^{n} \left(\frac{1}{2}k(\Delta x)^2 + \Delta x\right) = \frac{1}{2}(\Delta x)^2 \sum_{k=1}^{n} k + \Delta x \sum_{k=1}^{n} 1 = \frac{1}{2}\left(\frac{b^2}{n^2}\right)\left(\frac{n(n+1)}{2}\right) + \left(\frac{b}{n}\right)(n)$

$= \frac{1}{4}b^2\left(1 + \frac{1}{n}\right) + b \Rightarrow \int_0^b \left(\frac{x}{2} + 1\right)dx = \lim_{n\to\infty} \left(\frac{1}{4}b^2\left(1 + \frac{1}{n}\right) + b\right) = \frac{1}{4}b^2 + b$.

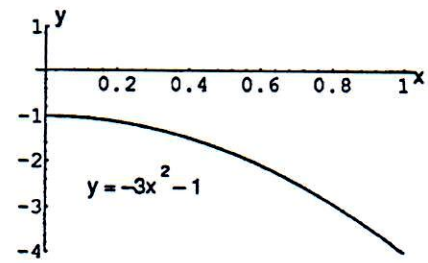
55. $\text{av}(f) = \left(\frac{1}{\sqrt{3}-0}\right) \int_0^{\sqrt{3}} (x^2 - 1)\, dx$
$= \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} x^2\, dx - \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} 1\, dx$
$= \frac{1}{\sqrt{3}} \left(\frac{(\sqrt{3})^3}{3}\right) - \frac{1}{\sqrt{3}} \left(\sqrt{3} - 0\right) = 1 - 1 = 0.$



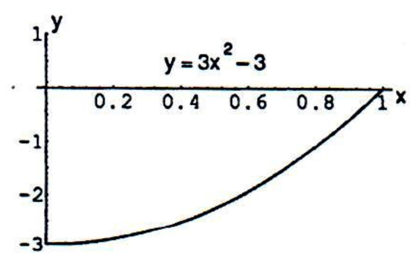
56. $\text{av}(f) = \left(\frac{1}{3-0}\right) \int_0^3 \left(-\frac{x^2}{2}\right) dx = \frac{1}{3}\left(-\frac{1}{2}\right) \int_0^3 x^2\, dx$
$= -\frac{1}{6}\left(\frac{3^3}{3}\right) = -\frac{3}{2};\ -\frac{x^2}{2} = -\frac{3}{2}.$



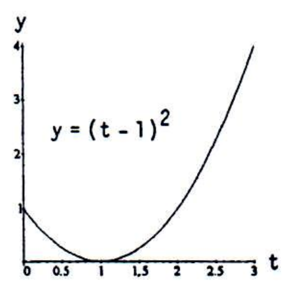
57. $\text{av}(f) = \left(\frac{1}{1-0}\right) \int_0^1 (-3x^2 - 1)\, dx =$
$= -3 \int_0^1 x^2\, dx - \int_0^1 1\, dx = -3\left(\frac{1^3}{3}\right) - (1 - 0)$
$= -2.$



58. $\text{av}(f) = \left(\frac{1}{1-0}\right) \int_0^1 (3x^2 - 3)\, dx =$
$= 3 \int_0^1 x^2\, dx - \int_0^1 3\, dx = 3\left(\frac{1^3}{3}\right) - 3(1 - 0)$
$= -2.$



59. $\text{av}(f) = \left(\frac{1}{3-0}\right) \int_0^3 (t - 1)^2\, dt$
$= \frac{1}{3} \int_0^3 t^2\, dt - \frac{2}{3} \int_0^3 t\, dt + \frac{1}{3} \int_0^3 1\, dt$
$= \frac{1}{3}\left(\frac{3^3}{3}\right) - \frac{2}{3}\left(\frac{3^2}{2} - \frac{0^2}{2}\right) + \frac{1}{3}(3 - 0) = 1.$



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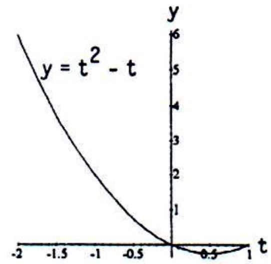
60. $\text{av}(f) = \left(\frac{1}{1-(-2)}\right)\int_{-2}^{1}(t^2 - t)\,dt$

$= \frac{1}{3}\int_{-2}^{1} t^2\,dt - \frac{1}{3}\int_{-2}^{1} t\,dt$

$= \frac{1}{3}\int_{0}^{1} t^2\,dt - \frac{1}{3}\int_{0}^{-2} t^2\,dt - \frac{1}{3}\left(\frac{1^2}{2} - \frac{(-2)^2}{2}\right)$

$= \frac{1}{3}\left(\frac{1^3}{3}\right) - \frac{1}{3}\left(\frac{(-2)^3}{3}\right) + \frac{1}{2} = \frac{3}{2}.$



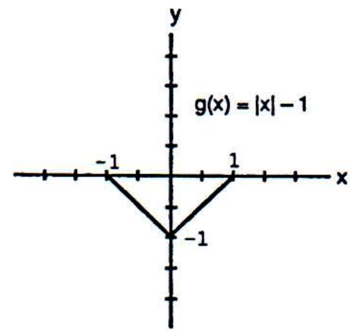
61. (a) $\text{av}(g) = \left(\frac{1}{1-(-1)}\right)\int_{-1}^{1}(|x| - 1)\,dx$

$= \frac{1}{2}\int_{-1}^{0}(-x-1)\,dx + \frac{1}{2}\int_{0}^{1}(x-1)\,dx$

$= -\frac{1}{2}\int_{-1}^{0} x\,dx - \frac{1}{2}\int_{-1}^{0} 1\,dx + \frac{1}{2}\int_{0}^{1} x\,dx - \frac{1}{2}\int_{0}^{1} 1\,dx$

$= -\frac{1}{2}\left(\frac{0^2}{2} - \frac{(-1)^2}{2}\right) - \frac{1}{2}(0-(-1)) + \frac{1}{2}\left(\frac{1^2}{2} - \frac{0^2}{2}\right) - \frac{1}{2}(1-0)$

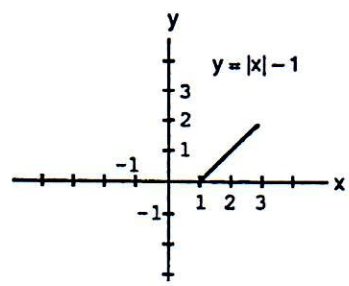
$= -\frac{1}{2}.$



(b) $\text{av}(g) = \left(\frac{1}{3-1}\right)\int_{1}^{3}(|x| - 1)\,dx = \frac{1}{2}\int_{1}^{3}(x-1)\,dx$

$= \frac{1}{2}\int_{1}^{3} x\,dx - \frac{1}{2}\int_{1}^{3} 1\,dx = \frac{1}{2}\left(\frac{3^2}{2} - \frac{1^2}{2}\right) - \frac{1}{2}(3-1)$

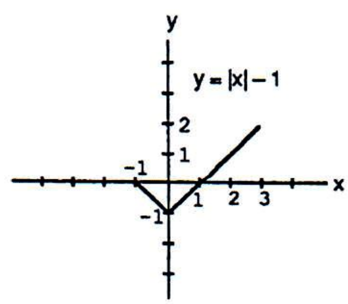
$= 1.$



(c) $\text{av}(g) = \left(\frac{1}{3-(-1)}\right)\int_{-1}^{3}(|x| - 1)\,dx$

$= \frac{1}{4}\int_{-1}^{1}(|x| - 1)\,dx + \frac{1}{4}\int_{1}^{3}(|x| - 1)\,dx$

$= \frac{1}{4}(-1+2) = \frac{1}{4}$ (see parts (a) and (b) above).



62. (a) $\text{av}(h) = \left(\frac{1}{0-(-1)}\right)\int_{-1}^{0} -|x|\,dx = \int_{-1}^{0} -(-x)\,dx$

$= \int_{-1}^{0} x\,dx = \frac{0^2}{2} - \frac{(-1)^2}{2} = -\frac{1}{2}.$