4.8 ANTIDERIVATIVES

1. (a) x^2 (b) $\frac{x^3}{3}$ (c) $\frac{x^3}{3}-x^2+x$

2. (a) $3x^2$ (b) $\frac{x^8}{8}$ (c) $\frac{x^8}{8}-3x^2+8x$

3. (a) x^{-3} (b) $-\frac{x^{-3}}{3}$ (c) $-\frac{x^{-3}}{3}+x^2+3x$

4. (a) $-x^{-2}$ (b) $-\frac{x^{-2}}{4}+\frac{x^3}{3}$ (c) $\frac{x^{-2}}{2}+\frac{x^2}{2}-x$

5. (a) $\frac{-1}{x}$ (b) $\frac{-5}{x}$ (c) $2x+\frac{5}{x}$

6. (a) $\frac{1}{x^2}$ (b) $\frac{-1}{4x^2}$ (c) $\frac{x^4}{4}+\frac{1}{2x^2}$

7. (a) $\sqrt{x^3}$ (b) $\sqrt{x}$ (c) $\frac{2}{3}\sqrt{x^3}+2\sqrt{x}$

8. (a) $x^{4/3}$ (b) $\frac{1}{2}x^{2/3}$ (c) $\frac{3}{4}x^{4/3}+\frac{3}{2}x^{2/3}$

9. (a) $x^{2/3}$ (b) $x^{1/3}$ (c) $x^{-1/3}$

10. (a) $x^{1/2}$ (b) $x^{-1/2}$ (c) $x^{-3/2}$

11. (a) $\ln|x|$ (b) $7\ln|x|$ (c) $x-5\ln|x|$

12. (a) $\frac{1}{3}\ln|x|$ (b) $\frac{2}{5}\ln|x|$ (c) $x+\frac{4}{3}\ln|x|+\frac{1}{x}$

13. (a) $\cos(\pi x)$ (b) $-3\cos x$ (c) $\frac{-\cos(\pi x)}{\pi}+\cos(3x)$

14. (a) $\sin(\pi x)$ (b) $\sin\left(\frac{\pi x}{2}\right)$ (c) $\left(\frac{2}{\pi}\right)\sin\left(\frac{\pi x}{2}\right)+\pi\sin x$

15. (a) $\tan x$ (b) $2\tan\left(\frac{x}{3}\right)$ (c) $-\frac{2}{3}\tan\left(\frac{3x}{2}\right)$

16. (a) $-\cot x$ (b) $\cot\left(\frac{3x}{2}\right)$ (c) $x+4\cot(2x)$

17. (a) $-\csc x$ (b) $\frac{1}{5}\csc(5x)$ (c) $2\csc\left(\frac{\pi x}{2}\right)$

18. (a) $\sec x$ (b) $\frac{4}{3}\sec(3x)$ (c) $\frac{2}{\pi}\sec\left(\frac{\pi x}{2}\right)$

19. (a) $\frac{1}{3}e^{3x}$ (b) $-e^{-x}$ (c) $2e^{x/2}$

20. (a) $-\frac{1}{2}e^{-2x}$ (b) $\frac{3}{4}e^{4x/3}$ (c) $-5e^{-x/5}$

21. (a) $\frac{1}{\ln 3}\cdot 3^x$ (b) $\frac{-1}{\ln 2}\cdot 2^{-x}$ (c) $\frac{1}{\ln(5/3)}\cdot\left(\frac{5}{3}\right)^x$

22. (a) $\frac{1}{\sqrt{3}+1}x^{\sqrt{3}+1}$ (b) $\frac{1}{\pi+1}x^{\pi+1}$ (c) $\frac{1}{\sqrt{2}}x^{\sqrt{2}}$

23. (a) $2\sin^{-1}x$ (b) $\frac{1}{2}\tan^{-1}x$ (c) $\frac{1}{2}\tan^{-1}(2x)$

24. (a) $\frac{1}{2}x^2-\frac{1}{\ln(1/2)}\cdot\left(\frac{1}{2}\right)^x$ (b) $\frac{1}{3}x^3+\frac{1}{\ln 2}\cdot 2^x$ (c) $\frac{1}{\ln\pi}\cdot\pi^x-\ln|x|$

25. $\int(x+1)\,dx=\frac{x^2}{2}+x+C$

26. $\int(5-6x)\,dx=5x-3x^2+C$

27. $\int\left(3t^2+\frac{t}{2}\right)dt=t^3+\frac{t^2}{4}+C$

28. $\int\left(\frac{t^2}{2}+4t^3\right)dt=\frac{t^3}{6}+t^4+C$

29. $\int(2x^3-5x+7)dx=\frac{1}{2}x^4-\frac{5}{2}x^2+7x+C$

30. $\int(1-x^2-3x^5)dx=x-\frac{1}{3}x^3-\frac{1}{2}x^6+C$

31. $\int\left(\frac{1}{x^2}-x^2-\frac{1}{3}\right)dx=\int\left(x^{-2}-x^2-\frac{1}{3}\right)dx=\frac{x^{-1}}{-1}-\frac{x^3}{3}-\frac{1}{3}x+C=-\frac{1}{x}-\frac{x^3}{3}-\frac{x}{3}+C$

32. $\int\left(\frac{1}{5}-\frac{2}{x^3}+2x\right)dx=\int\left(\frac{1}{5}-2x^{-3}+2x\right)dx=\frac{1}{5}x-\left(\frac{2x^{-2}}{-2}\right)+\frac{2x^2}{2}+C=\frac{x}{5}+\frac{1}{x^2}+x^2+C$

33. $\int x^{-1/3}dx=\frac{x^{2/3}}{\frac{2}{3}}+C=\frac{3}{2}x^{2/3}+C$

34. $\int x^{-5/4}dx=\frac{x^{-1/4}}{-\frac{1}{4}}+C=\frac{-4}{\sqrt[4]{x}}+C$

35. $\int\left(\sqrt{x}+\sqrt[3]{x}\right)dx=\int\left(x^{1/2}+x^{1/3}\right)dx=\frac{x^{3/2}}{\frac{3}{2}}+\frac{x^{4/3}}{\frac{4}{3}}+C=\frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$

36. $\int\left(\frac{\sqrt{x}}{2}+\frac{2}{\sqrt{x}}\right)dx=\int\left(\frac{1}{2}x^{1/2}+2x^{-1/2}\right)dx=\frac{1}{2}\left(\frac{x^{3/2}}{\frac{3}{2}}\right)+2\left(\frac{x^{1/2}}{\frac{1}{2}}\right)+C=\frac{1}{3}x^{3/2}+4x^{1/2}+C$

37. $\int\left(8y-\frac{2}{y^{1/4}}\right)dy=\int\left(8y-2y^{-1/4}\right)dy=\frac{8y^2}{2}-2\left(\frac{y^{3/4}}{\frac{3}{4}}\right)+C=4y^2-\frac{8}{3}y^{3/4}+C$

38. $\int\left(\frac{1}{7}-\frac{1}{y^{5/4}}\right)dy=\int\left(\frac{1}{7}-y^{-5/4}\right)dy=\frac{1}{7}y-\left(\frac{y^{-1/4}}{-\frac{1}{4}}\right)+C=\frac{y}{7}+\frac{4}{y^{1/4}}+C$

39. $\int 2x\left(1-x^{-3}\right)dx=\int\left(2x-2x^{-2}\right)dx=\frac{2x^2}{2}-2\left(\frac{x^{-1}}{-1}\right)+C=x^2+\frac{2}{x}+C$

40. $\int x^{-3}(x+1)\,dx=\int\left(x^{-2}+x^{-3}\right)dx=\frac{x^{-1}}{-1}+\left(\frac{x^{-2}}{-2}\right)+C=-\frac{1}{x}-\frac{1}{2x^2}+C$

41. $\int\frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt=\int\left(\frac{t^{3/2}}{t^2}+\frac{t^{1/2}}{t^2}\right)dt=\int\left(t^{-1/2}+t^{-3/2}\right)dt=\frac{t^{1/2}}{\frac{1}{2}}+\left(\frac{t^{-1/2}}{-\frac{1}{2}}\right)+C=2\sqrt{t}-\frac{2}{\sqrt{t}}+C$

42. $\int \frac{4+\sqrt{t}}{t^3}\,dt = \int\left(\frac{4}{t^3}+\frac{t^{1/2}}{t^3}\right)dt = \int\left(4t^{-3}+t^{-5/2}\right)dt = 4\left(\frac{t^{-2}}{-2}\right)+\left(\frac{t^{-3/2}}{-\frac{3}{2}}\right)+C = -\frac{2}{t^2}-\frac{2}{3t^{3/2}}+C$

43. $\int -2\cos t\,dt = -2\sin t + C$

44. $\int -5\sin t\,dt = 5\cos t + C$

45. $\int 7\sin\frac{\theta}{3}\,d\theta = -21\cos\frac{\theta}{3}+C$

46. $\int 3\cos 5\theta\,d\theta = \frac{3}{5}\sin 5\theta + C$

47. $\int -3\csc^2 x\,dx = 3\cot x + C$

48. $\int -\frac{\sec^2 x}{3}\,dx = -\frac{\tan x}{3}+C$

49. $\int \frac{\csc\theta\cot\theta}{2}\,d\theta = -\frac{1}{2}\csc\theta + C$

50. $\int \frac{2}{5}\sec\theta\tan\theta\,d\theta = \frac{2}{5}\sec\theta + C$

51. $\int (e^{3x}+5e^{-x})dx = \frac{e^{3x}}{3}-5e^{-x}+C$

52. $\int (2e^x-3e^{-2x})dx = 2e^x+\frac{3}{2}e^{-2x}+C$

53. $\int (e^{-x}+4^x)dx = -e^{-x}+\frac{4^x}{\ln 4}+C$

54. $\int (1.3)^x\,dx = \frac{(1.3)^x}{\ln(1.3)}+C$

55. $\int (4\sec x\tan x - 2\sec^2 x)\,dx = 4\sec x - 2\tan x + C$

56. $\int \frac{1}{2}(\csc^2 x - \csc x\cot x)\,dx = -\frac{1}{2}\cot x + \frac{1}{2}\csc x + C$

57. $\int (\sin 2x - \csc^2 x)\,dx = -\frac{1}{2}\cos 2x + \cot x + C$

58. $\int (2\cos 2x - 3\sin 3x)\,dx = \sin 2x + \cos 3x + C$

59. $\int \frac{1+\cos 4t}{2}\,dt = \int\left(\frac{1}{2}+\frac{1}{2}\cos 4t\right)dt = \frac{1}{2}t+\frac{1}{2}\left(\frac{\sin 4t}{4}\right)+C = \frac{t}{2}+\frac{\sin 4t}{8}+C$

60. $\int \frac{1-\cos 6t}{2}\,dt = \int\left(\frac{1}{2}-\frac{1}{2}\cos 6t\right)dt = \frac{1}{2}t-\frac{1}{2}\left(\frac{\sin 6t}{6}\right)+C = \frac{t}{2}-\frac{\sin 6t}{12}+C$

61. $\int\left(\frac{1}{x}-\frac{5}{x^2+1}\right)dx = \ln|x| - 5\tan^{-1}x + C$

62. $\int\left(\frac{2}{\sqrt{1-y^2}}-\frac{1}{y^{1/4}}\right)dy = 2\sin^{-1}y - \frac{4}{3}y^{3/4}+C$

63. $\int 3x^{\sqrt{3}}\,dx = \frac{3x^{(\sqrt{3}+1)}}{\sqrt{3}+1}+C$

64. $\int x^{(\sqrt{2}-1)}\,dx = \frac{x^{\sqrt{2}}}{\sqrt{2}}+C$

65. $\int (1+\tan^2\theta)\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$

66. $\int (2+\tan^2\theta)\,d\theta = \int (1+1+\tan^2\theta)\,d\theta = \int (1+\sec^2\theta)\,d\theta = \theta+\tan\theta+C$

67. $\int \cot^2 x\,dx = \int (\csc^2 x - 1)\,dx = -\cot x - x + C$

68. $\int (1-\cot^2 x)\,dx = \int (1-(\csc^2 x - 1))\,dx = \int (2-\csc^2 x)\,dx = 2x + \cot x + C$

69. $\int \cos\theta(\tan\theta+\sec\theta)\,d\theta = \int (\sin\theta+1)\,d\theta = -\cos\theta+\theta+C$

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70. $\int \frac{\csc\theta}{\csc\theta-\sin\theta}\,d\theta = \int\left(\frac{\csc\theta}{\csc\theta-\sin\theta}\right)\left(\frac{\sin\theta}{\sin\theta}\right)\,d\theta = \int \frac{1}{1-\sin^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta}\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$

71. $\frac{d}{dx}\left(\frac{(7x-2)^4}{28}+C\right) = \frac{4(7x-2)^3(7)}{28} = (7x-2)^3$

72. $\frac{d}{dx}\left(-\frac{(3x+5)^{-1}}{3}+C\right) = -\left(-\frac{(3x+5)^{-2}(3)}{3}\right) = (3x+5)^{-2}$

73. $\frac{d}{dx}\left(\frac{1}{5}\tan(5x-1)+C\right) = \frac{1}{5}(\sec^2(5x-1))(5) = \sec^2(5x-1)$

74. $\frac{d}{dx}\left(-3\cot\left(\frac{x-1}{3}\right)+C\right) = -3\left(-\csc^2\left(\frac{x-1}{3}\right)\right)\left(\frac{1}{3}\right) = \csc^2\left(\frac{x-1}{3}\right)$

75. $\frac{d}{dx}\left(\frac{-1}{x+1}+C\right) = (-1)(-1)(x+1)^{-2} = \frac{1}{(x+1)^2}$

76. $\frac{d}{dx}\left(\frac{x}{x+1}+C\right) = \frac{(x+1)(1)-x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$

77. $\frac{d}{dx}(\ln|x+1|+C) = \frac{1}{x+1}$

78. $\frac{d}{dx}(xe^x - e^x + C) = x\cdot e^x + (1)\cdot e^x - e^x = xe^x$

79. $\frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C\right) = \frac{1}{a}\cdot\frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{d}{dx}\left(\frac{x}{a}\right) = \frac{1}{a^2\left(1+\frac{x^2}{a^2}\right)} = \frac{1}{a^2+x^2}$

80. $\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)+C\right) = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\frac{d}{dx}\left(\frac{x}{a}\right) = \frac{1}{a\sqrt{1-\left(\frac{x}{a}\right)^2}} = \frac{1}{\sqrt{a^2-x^2}}$

81. If $y = \ln x - \frac{1}{2}\ln(1+x^2) - \frac{\tan^{-1}x}{x} + C$, then

$$dy = \left[\frac{1}{x} - \frac{x}{1+x^2} - \frac{\left(\frac{x}{1+x^2}\right)-\tan^{-1}x}{x^2}\right]dx = \left(\frac{1}{x} - \frac{x}{1+x^2} - \frac{1}{x(1+x^2)} + \frac{\tan^{-1}x}{x^2}\right)dx = \frac{x(1+x^2)-x^3-x+(\tan^{-1}x)(1+x^2)}{x^2(1+x^2)}\,dx = \frac{\tan^{-1}x}{x^2}\,dx,$$

which verifies the formula

82. If $y = x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\sin^{-1}x + C$, then

$dy = \left[(\sin^{-1}x)^2 + \frac{2x(\sin^{-1}x)}{\sqrt{1-x^2}} - 2 + \frac{-2x}{\sqrt{1-x^2}}\sin^{-1}x + 2\sqrt{1-x^2}\left(\frac{1}{\sqrt{1-x^2}}\right)\right]dx = (\sin^{-1}x)^2\,dx$, which verifies the formula

83. (a) Wrong: $\frac{d}{dx}\left(\frac{x^2}{2}\sin x + C\right) = \frac{2x}{2}\sin x + \frac{x^2}{2}\cos x = x\sin x + \frac{x^2}{2}\cos x \neq x\sin x$

(b) Wrong: $\frac{d}{dx}(-x\cos x + C) = -\cos x + x\sin x \neq x\sin x$

(c) Right: $\frac{d}{dx}(-x\cos x + \sin x + C) = -\cos x + x\sin x + \cos x = x\sin x$

84. (a) Wrong: $\frac{d}{d\theta}\left(\frac{\sec^3\theta}{3}+C\right) = \frac{3\sec^2\theta}{3}(\sec\theta\tan\theta) = \sec^3\theta\tan\theta \neq \tan\theta\sec^2\theta$

(b) Right: $\frac{d}{d\theta}\left(\frac{1}{2}\tan^2\theta + C\right) = \frac{1}{2}(2\tan\theta)\sec^2\theta = \tan\theta\sec^2\theta$

(c) Right: $\frac{d}{d\theta}\left(\frac{1}{2}\sec^2\theta + C\right) = \frac{1}{2}(2\sec\theta)\sec\theta\tan\theta = \tan\theta\sec^2\theta$