Exercises 4.7 Solutions to selected problems

(2) $f(x) = x^3+3x+1$

$f'(x) = 3x^2+3$

$x_0 = 0$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0 - \frac{1}{3}$

$= -\frac{1}{3}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= -\frac{29}{90}$

(4) $f(x) = 2x - x^2+1$

$f'(x) = 2 - 2x$

$x_0 = 0$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0 - \frac{f(0)}{f'(0)}$

$= -\frac{1}{2}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= -\frac{5}{12}$

$x_0 = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{f(2)}{f'(2)}$

$= \frac{5}{2}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= \frac{29}{12}$

(6) $f(x) = x^4 - 2$, $f'(x) = 4x^3$

$x_0 = -1$, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{f(-1)}{f'(-1)} = -\frac{5}{4}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -\frac{5}{4} - \frac{f(-\frac{5}{4})}{f'(\frac{5}{4})} = \frac{-2387}{2000} \approx -1.1935$

Note that $f(x_2) \approx 0.0290$ so, it's good up to first decimal place.

(8) We have to choose x_0 that is close to $\frac{\pi}{2}$, such as 1.5 or 1.6. If we choose x_0 away from $\frac{\pi}{2}$, we will get other roots of the equation $\cos x = 0$. For example, if we choose $x_0 = 4.5$, we will get x_n close to $\frac{3\pi}{2}$ because $\frac{3\pi}{2}$ is also a root of $\cos x = 0$.

(20) Let $f(x) = \cos x - (-x) = \cos x + x$

$f'(x) = \sin x - 1$

Note that $f(0) = \cos 0 + 0 = 1 > 0$

$f(-\pi) = \cos(-\pi) + (-\pi) = -1 - \pi < 0$

By intermediat value theorem, there is a solution of $f(x) = 0$ in $[-\pi, 0]$.

Let $x_0 = 0$, then $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 0 - \dfrac{f(0)}{f'(0)} = -1$

Note that $f(-1) = \cos(-1) - 1 \approx -0.459698$, so x_1 is still not good enough.

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = -1 - \dfrac{f(-1)}{f'(-1)} \approx -0.750364$

Note that $f(x_2) \approx -0.01892$, so we get only the first decimal place

$x_3 = x_2 - \dfrac{f(x_1)}{f'(x_1)} \approx -0.739113$

Note that $f(x_3) \approx -0.0000466$, so this is good enough because we get accuracy upto 4 decimal places.

So, $\cos x = -x$ at $x \approx -0.739113$