Exercises 3.10 Solutions to selected problems

20. r = radius

A = Area

Know $\frac{dr}{dt} = 0.01$ cm/min.

Want to find $\frac{dA}{dt}$ when $r = 50$ cm.

so, we need to find relation between $\frac{dr}{dt}$ and $\frac{dA}{dt}$.

$\Rightarrow$ Start with relation between r and A.

Area formula $A = \pi r^2$

Then, Differentiate both sides with respect to t

we get $$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Plug-in $\frac{dr}{dt} = 0.01$ and $r = 50$ cm, we get

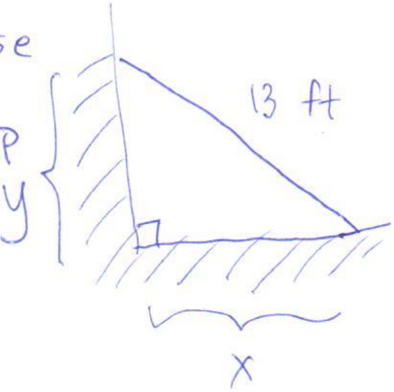
$$\frac{dA}{dt} = 2\pi (50)(0.01)$$

so, $$\frac{dA}{dt} = \pi \text{ cm/min}$$

$$\frac{dA}{dt} \approx 3.14 \text{ cm/min}$$

(23) (a) x = distance from the corner to the base

y = distance from the corner to the top



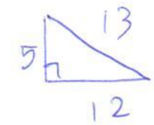
So, in this case, we have that

$\frac{dx}{dt}$ = the speed at which the base is moving away from the corner.

$\frac{dy}{dt}$ = the speed at which the top is moving down.

Know $\frac{dx}{dt} = 5$ ft/sec.

Want to find $\frac{dy}{dt}$ when $x = 12$ ft.

(Note that when $x = 12$, we have $y = 5$ )

So, ~~we~~ we need to find relation between $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$\Rightarrow$ start with relation between x and y.

<u>Pythagorean theorem</u> $x^2 + y^2 = 13^2$

Then, differentiate both sides with respect to t

we get $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Plug-in : $(12)(5) + (5)\frac{dy}{dt} = 0$

$$\frac{dy}{dt} = \frac{-(12)(5)}{5} = -12 \text{ ft/sec.}$$

(39)

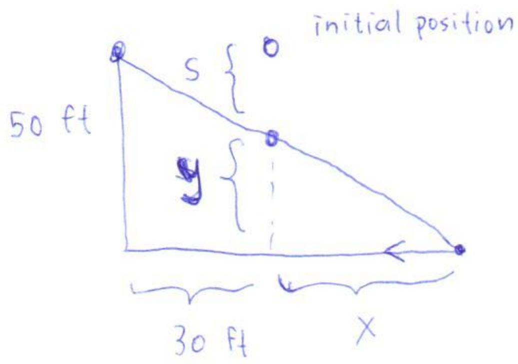
x = distance from the shadow to the dropping point

y = height of ball

Note that $y = 50 - s$

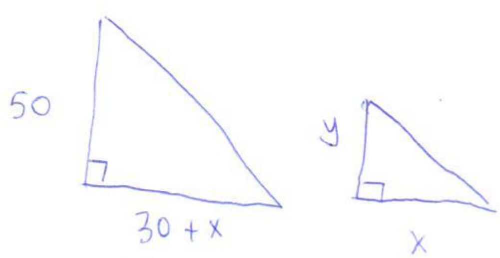
$y(t) = 50 - 16t^2$

so $\frac{dy}{dy} = -32t$



Know $\frac{dy}{dt}$, want to find $\frac{dx}{dt}$ after $\frac{1}{2}$ second.

(After $\frac{1}{2}$ second, $y = 50 - 16\left(\frac{1}{2}\right)^2 = 46$ ft.)



From similar triangles, we have

$$\frac{x}{30+x} = \frac{y}{50}$$

$$50x = 30y + xy$$

Differentiate both sides with respect to t

$$50\frac{dx}{dt} = 30\frac{dy}{dt} + x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{x}{30+x} = \frac{y}{50}$$

When $y = 46$, we get

$$\frac{x}{30+x} = \frac{46}{50}$$

$$\frac{x}{30+x} = \frac{23}{25}$$

$$25x = 690 + 23x$$

$$2x = 690$$

$$x = 345 \text{ ft}$$

$\Rightarrow$

At $t = \frac{1}{2}$, we have $y = 46$, $\frac{dy}{dt} = -16$ and $x = 345$.

Plug-in

$$50\frac{dx}{dt} = 30(-16) + 345(-16) + 46\frac{dx}{dt}$$

$$4\frac{dx}{dt} = 375(-16)$$

$$\frac{dx}{dt} = -1500 \text{ ft/sec}$$