Exercises 3.8 — Solutions to selected problems

(8) $f(x) = x^2 - 4x - 5, \quad x > 2.$ $\Rightarrow f'(x) = 2x-4$

$x = 0 = f(5)$, so $f^{-1}(0) = 5$

$$\left.\frac{df^{-1}}{dx}\right|_{x=0} = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(5)} = \frac{1}{2(5)-4} = \frac{1}{6}$$

(12) $y = \ln kx \Rightarrow y' = \frac{1}{kx}\cdot k = \frac{1}{x}$

(14) $y = \ln t^{3/2} = \frac{3}{2}\ln t \Rightarrow y' = \frac{3}{2}\cdot\frac{1}{t}$

(16) $y = \ln\frac{10}{x} = \ln 10 - \ln x \Rightarrow y' = 0 - \frac{1}{x} = -\frac{1}{x}$

(18) $y = \ln(2\theta+2) \Rightarrow y' = \frac{1}{2\theta+2}\cdot 2 = \frac{2}{2\theta+2} = \frac{1}{\theta+1}$

(20) $y = (\ln x)^3 \Rightarrow y' = 3(\ln x)^2\cdot(\ln x)' = 3(\ln x)^2\cdot\frac{1}{x}$

(22) $y = t\sqrt{\ln t} \Rightarrow y' = t\cdot\frac{1}{2\sqrt{\ln t}}\cdot\frac{1}{t} + \sqrt{\ln t}\cdot 1 = \frac{1}{2\sqrt{\ln t}} + \sqrt{\ln t}$

(24) $y = (x^2\ln x)^4 \Rightarrow$
$$\begin{aligned} y' &= 4(x^2\ln x)^3\cdot(x^2\ln x)' \\ &= 4(x^2\ln x)^3\cdot\left[x^2\cdot\frac{1}{x} + \ln x\cdot 2x\right] \\ &= 4(x^2\ln x)^3\cdot[x + 2x\ln x] \end{aligned}$$

(26) $y = \frac{1+\ln t}{t} \Rightarrow y' = \frac{t(1+\ln t)' - (1+\ln t)(t)'}{t^2}$

$$y' = \frac{t\cdot\left(\frac{1}{t}\right) - (1+\ln t)\cdot 1}{t}$$

$$y' = \frac{1-(1+\ln t)}{t} = \frac{-\ln t}{t}$$

(28) $y = \dfrac{x\ln x}{1+\ln x}$

$y' = \dfrac{(1+\ln x)(x\ln x)' - (x\ln x)(1+\ln x)'}{(1+\ln x)^2}$

$y' = \dfrac{(1+\ln x)(\ln x + 1) - x\ln x \cdot \left(\frac{1}{x}\right)}{(1+\ln x)^2}$

(30) $y = \ln(\ln(\ln x))$

$y' = \dfrac{1}{\ln(\ln x)} \cdot [\ln(\ln x)]' = \dfrac{1}{\ln(\ln x)} \cdot \dfrac{1}{\ln x} \cdot [\ln x]'$

$= \dfrac{1}{\ln(\ln x)} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}$

(32) $y = \ln(\sec\theta + \tan\theta)$

$y' = \dfrac{1}{\sec\theta + \tan\theta} \cdot (\sec\theta\tan\theta + \sec^2\theta)$

$= \dfrac{\sec\theta(\tan\theta + \sec\theta)}{\sec\theta + \tan\theta} = \sec\theta$

(34) $y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \frac{1}{2}\ln(1+x) - \frac{1}{2}\ln(1-x)$

$y' = \frac{1}{2} \cdot \frac{1}{1+x} - \frac{1}{2}\frac{1}{1-x}(-1)$

(36) $y = \sqrt{\ln\sqrt{t}} = \sqrt{\frac{1}{2}\ln t}$

$y' = \dfrac{1}{2\sqrt{\frac{1}{2}\ln t}} \cdot \left(\frac{1}{2}\ln t\right)' = \dfrac{1}{2\sqrt{\frac{1}{2}\ln t}} \cdot \frac{1}{2} \cdot \frac{1}{t} = \dfrac{1}{4t\sqrt{\frac{1}{2}\ln t}}$

(38) $y = \ln\left(\dfrac{\sqrt{\sin\theta\cos\theta}}{1+2\ln\theta}\right)$ see quiz solution

(40) see quiz solution

(42) $y = \sqrt{(x^2+1)(x-1)^2}$

$$\ln y = \frac{1}{2}\ln(x^2+1) + \ln(x-1)$$

$$\frac{1}{y}y' = \frac{1}{2}\cdot\frac{1}{x^2+1}(2x) + \frac{1}{x-1}(1) = \frac{x}{x^2+1} + \frac{1}{x-1}$$

$$y' = y\left[\frac{x}{x^2+1} + \frac{1}{x-1}\right] = \sqrt{(x^2+1)(x-1)^2}\left[\frac{x}{x^2+1} + \frac{1}{x-1}\right]$$

(44) $y = \sqrt{\frac{1}{t(t+1)}}$

$$\ln y = -\frac{1}{2}\ln t - \frac{1}{2}\ln(t+1)$$

$$\frac{1}{y}y' = -\frac{1}{2}\cdot\frac{1}{t} - \frac{1}{2}\cdot\frac{1}{t+1}\cdot 1 = -\frac{1}{2t} - \frac{1}{2(t+1)}$$

$$y' = y\left[-\frac{1}{2t} - \frac{1}{2(t+1)}\right] = \sqrt{\frac{1}{t(t+1)}}\left[-\frac{1}{2t} - \frac{1}{2(t+1)}\right]$$

(46) $y = (\tan\theta)\sqrt{2\theta+1}$

$$\ln y = \ln(\tan\theta) + \frac{1}{2}\ln(2\theta+1)$$

$$\frac{1}{y}y' = \frac{1}{\tan\theta}\sec^2\theta + \frac{1}{2}\cdot\frac{1}{2\theta+1}\cdot 2$$

$$y' = y\left[\frac{\sec^2\theta}{\tan\theta} + \frac{1}{2\theta+1}\right]$$

(48) $\ln y = -\ln t - \ln(t+1) - \ln(t+2)$

$$\frac{1}{y}y' = -\frac{1}{t} - \frac{1}{t+1} - \frac{1}{t+2}$$

$$y' = y\left[-\frac{1}{t} - \frac{1}{t+1} - \frac{1}{t+2}\right]$$

(50) $\ln y = \ln\theta + \ln(\sin\theta) - \frac{1}{2}\ln(\sec\theta)$

$$\frac{1}{y}y' = \frac{1}{\theta} + \frac{1}{\sin\theta}\cdot\cos\theta - \frac{1}{2}\cdot\frac{1}{\sec\theta}\sec\theta\tan\theta$$

$$y' = y\left[\frac{1}{\theta} + \cot\theta - \frac{1}{2}\tan\theta\right]$$

(52) $\ln y = \frac{1}{2}\ln\left(\frac{(x+1)^{10}}{(2x+1)^5}\right) = \frac{10}{2}\ln(x+1) - \frac{5}{2}\ln(2x+1)$

$$\frac{1}{y}y' = 5\cdot\frac{1}{x+1} - \frac{5}{2}\cdot\frac{1}{2x+1}\cdot 2$$

$$y' = y\left[\frac{5}{x+1} - \frac{5}{2x+1}\right]$$

(54) $\ln y = \frac{1}{3}\ln x + \frac{1}{3}\ln(x+1) + \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln(x^2+1) - \frac{1}{3}\ln(2x+3)$

$$y' = y\left[\frac{1}{3x} + \frac{1}{3(x+1)} + \frac{1}{3(x-2)} - \frac{1}{3(x^2+1)}\cdot(2x) - \frac{1}{3(2x+3)}\cdot 2\right]$$

(56) $y = \ln(3\theta e^{-\theta}) = \ln 3 + \ln\theta + \ln e^{-\theta} = \ln 3 + \ln\theta - \theta\ln e$
$= \ln 3 + \ln\theta - \theta$

$$\Rightarrow y' = 0 + \frac{1}{\theta} - 1 = \frac{1}{\theta} - 1$$

(58) $y = \ln(2e^{-t}\sin t) = \ln 2 + \ln e^{-t} + \ln(\sin t)$
$= \ln 2 - t + \ln(\sin t)$

$$\Rightarrow y' = 0 - 1 + \frac{1}{\sin t}(\cos t) = -1 + \cot t$$

(60) see quiz solution

(62) $y = e^{\sin t}(\ln t^2 + 1) = e^{\sin t}(2\ln t + 1)$

$y' = e^{\sin t}\left(2 \cdot \frac{1}{t} + 0\right) + (2\ln t + 1)\, e^{\sin t} \cdot \cos t$

$= e^{\sin t}\left(\frac{2}{t}\right) + (2\ln t + 1)\, e^{\sin t} \cos t$

(64) $\ln xy = e^{x+y}$

$\ln x + \ln y = e^{x+y}$

$\frac{1}{x} + \frac{1}{y} y' = e^{x+y}(1 + y')$

$\frac{1}{y} y' - e^{x+y} y' = e^{x+y} - \frac{1}{x}$

$\left(\frac{1}{y} - e^{x+y}\right) y' = e^{x+y} - \frac{1}{x}$

$y' = \dfrac{e^{x+y} - \frac{1}{x}}{\frac{1}{y} - e^{x+y}}$

(66) $\tan y = e^x + \ln x$

$\sec^2 y \cdot y' = e^x + \frac{1}{x}$

$y' = \dfrac{e^x + \frac{1}{x}}{\sec^2 y}$

(68) $y = 3^{-x} \Rightarrow y' = 3^{-x} \ln 3 \cdot (-1)$

(70) $y = 2^{(s^2)} \Rightarrow y' = 2^{(s^2)} \ln 2\, (s^2)' = 2s \cdot 2^{(s^2)} \cdot \ln 2$

(72) $y = t^{1-e} \Rightarrow y' = (1-e)\, t^{1-e-1} = (1-e)\, t^{-e}$

(74) $y = \log_3(1 + \theta \ln 3)$

$y' = \frac{1}{\ln 3(1 + \theta \ln 3)} (1 + \theta \ln 3)' = \frac{\ln 3}{\ln 3(1 + \theta \ln 3)} = \frac{1}{1 + \theta \ln 3}$

76) $y = \log_{25} e^x - \log_5 \sqrt{x} = x\log_{25} e - \frac{1}{2}\log_5 x$

$y' = \log_{25} e - \frac{1}{2} \cdot \frac{1}{\ln 5\, x}$

78) $y = \log_3 r \cdot \log_9 r$

$y' = \log_3 r \cdot \frac{1}{r\ln 9} + \log_9 r \cdot \frac{1}{r\ln 3}$

80) $y = \ln 5\left[\log_5 (7x) - \log_5 (3x+2)\right]$

$y' = \ln 5\left[\frac{1}{7x\ln 5}\cdot 7 - \frac{1}{(3x+2)\ln 5}\cdot 3\right]$

82) $y = \log_7 \sin\theta + \log_7 \cos\theta - \log_7 e^\theta - \log_7 2^\theta$

$= \log_7 \sin\theta + \log_7 \cos\theta - \theta\log_7 e - \theta\log_7 2$

$y' = \frac{1}{\sin\theta \cdot \ln 7}\cos\theta + \frac{1}{\cos\theta\ln 7}(-\sin\theta) - \log_7 e - \log_7 2$

84) $y = \log_2 x^2 + \log_2 e^2 - \log_2 2 - \frac{1}{2}\log_2 (x+1)$

$= 2\log_2 x + 2\log_2 e - 1 - \frac{1}{2}\log_2 (x+1)$

$y' = 2\frac{1}{x\ln 2} + 0 - 0 - \frac{1}{2}\cdot\frac{1}{(x+1)\ln 2}$

$= \frac{2}{x\ln 2} - \frac{1}{2(x+1)\ln 2}$

86 $y = 3\log_8(\log_2 t)$

$$y' = 3 \cdot \frac{1}{\log_2 t \cdot \ln 8} (\log_2 t)' = \frac{3}{(\log_2 t)(\ln 8)} \cdot \frac{1}{t \ln 2}$$

88 $y = t \log_3 (e^{(\sin t)(\ln 3)})$

$$= t\left[(\sin t)(\ln 3) \cdot \log_3 e\right]$$

$$= \underbrace{(\ln 3) \cdot (\log_3 e)}_{\text{constant}} \cdot t \sin t$$

$$y' = \ln 3 \cdot \log_3 e \cdot \left[t \cos t + \sin t \cdot 1\right]$$

90 $y = x^{(x+1)}$ see quiz solution

92 $y = t^{\sqrt{t}} \Rightarrow \ln y = \ln t^{\sqrt{t}}$

$$\Rightarrow \ln y = \sqrt{t} \cdot \ln t$$

$$\frac{1}{y} y' = \sqrt{t} \cdot \frac{1}{t} + \ln t \cdot \frac{1}{2\sqrt{t}}$$

$$y' = y\left[\frac{\sqrt{t}}{t} + \frac{\ln t}{2\sqrt{t}}\right]$$

94 $y = x^{\sin x}$ see quiz solution

96 $y = (\ln x)^{\ln x}$

$$\ln y = \ln (\ln x)^{\ln x}$$

$$\ln y = \ln x \cdot [\ln(\ln x)]$$

$$\frac{1}{y} y' = \ln x \cdot \left[\frac{1}{\ln x} \cdot (\ln x)' + \ln(\ln x) \cdot \frac{1}{x} \right]$$

$$y' = y\left[\ln x \left(\frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) \cdot \frac{1}{x} \right)\right]$$