

### Exercises 3.3

first derivatives

$$1. y' = -2x$$

$$2. y' = 2x + 1$$

$$3. s' = 15t^2 - 15t^6$$

$$4. w' = 21z^6 - 21z^2 + 42z$$

$$5. y' = 4x^2 - 1 + 2e^x$$

$$6. y' = x^2 + x + \frac{1}{x}$$

$$7. w' = -6z^{-3} + z^{-2}$$

$$8. s' = 2t^2 - 8t^{-3}$$

$$9. y' = 12x - 10 + 10x^{-3}$$

$$10. y' = -2 + x^{-4}$$

$$11. r' = -\frac{2}{3}s^{-3} + \frac{5}{2}s^{-2}$$

$$12. r' = -12\theta^{-2} + 12\theta^{-6} - 64\theta^{-5}$$

second derivatives

$$y'' = -2$$

$$y'' = 2$$

$$s'' = 30t - 60t^5$$

$$w'' = 126z^5 - 42z + 42$$

$$y'' = 8x + 2e^x$$

$$y'' = 2x + \frac{1}{x^2}$$

$$w'' = 18z^{-4} - 2z^{-3}$$

$$s'' = -4t^{-3} + 24t^{-4}$$

$$y'' = 12 - 30x^{-4}$$

$$y'' = -4x^{-5}$$

$$r'' = 2s^{-4} - 5s^{-3}$$

$$r'' = 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$$

$$13. (a) y' = -2x(x^2 - x + 1) + (3 - x^2)(2x^2 - 1)$$

$$= -2x^4 + 2x^2 - 2x + 6x^2 - 3 - 2x^4 + x^2$$

$$= -5x^4 + 12x^2 - 2x - 3$$

$$(b) y = 3x^3 - 3x + 3 - x^5 + x^3 - x^2$$

$$= -x^5 + 4x^3 - x^2 - 3x + 3$$

$$14. (a) y' = 2(5x^2 - 6x) + (12x + 3)(10x - 4)$$

$$= 10x^2 - 8x + 20x^2 - 8x + 30x^2 + 30x - 12$$

$$= 30x^2 + 16x - 12$$

$$(b) y = 10x^3 - 8x^2 + 15x^2 - 12x$$

$$= 10x^3 + 7x^2 - 12x$$

$$15. (a) y' = 2x(x + 5 + \frac{1}{x}) + (x^2 + 1)(10 - x^{-2})$$

$$= 2x^2 + 10x + 2 + x^2 - 1 + 10 - x^{-2}$$

$$= 3x^2 + 10x - x^{-2} + 12$$

$$(b) y = x^2 + 5x^2 + x + x + 5 + \frac{1}{x}$$

$$= x^2 + 5x^2 + 2x + \frac{1}{x} + 5$$

$$16. (a) y' = 2x(x^{\frac{3}{4}} - x^{-3}) + (4x^{\frac{3}{4}})(\frac{3}{4}x^{-\frac{1}{4}} + 3x^{-4})$$

$$= 2x^{\frac{7}{4}} - 2x^{-2} + \frac{3}{4}x^{\frac{3}{4}} + 3x^{-4} + \frac{3}{4}x^{\frac{3}{4}} + 3x^{-2}$$

$$= \frac{11}{4}x^{\frac{3}{4}} + \frac{3}{4}x^{-\frac{1}{4}} + 3x^{-4} + x^{-2}$$

$$(b) y = x^{\frac{3}{4}} - x^{-3} + x^{\frac{11}{4}} - x^{-1}$$

$$17. y' = \frac{2(3x-1) - 3(2x+5)}{(3x-2)^2} = \frac{6x-4-6x-15}{9x^2-12x+4} = -\frac{19}{9x^2-12x+4}$$

$$18. z' = \frac{-3(3x^2+x) - (4-2x)(6x+1)}{(3x^2+x)^2} = \frac{-9x^2-3x-24x-4+18x^2+2x}{9x^4+6x^3+x^2} = \frac{9x^2-24x-4}{9x^4+6x^3+x^2}$$

$$19. g(x) = 2 \frac{x^2-4}{2x+1}$$

$$g'(x) = -2 \frac{2x(2x+1) - 2(x^2-4)}{(2x+1)^2} = -2 \frac{4x^2+2x-2x^2+8}{(2x+1)^2} = -2 \frac{2x^2+x+4}{(2x+1)^2}$$

$$20. f(t) = \frac{(t+1)(t-1)}{(t+2)(t-1)} = \frac{t+1}{t+2}$$

$$f'(t) = \frac{t+2-t-1}{(t+2)^2} = \frac{1}{(t+2)^2}$$

$$21. v = \frac{1-t}{t+1}$$

$$v' = \frac{-(t+1) - 2t(1-t)}{(t+1)^2} = \frac{-t-1-2t+2t^2}{(t+1)^2} = \frac{2t^2-3t-1}{(t+1)^2}$$

$$22. w = \frac{x+5}{2x-7}$$

$$w' = \frac{2x-7-2(x+5)}{(2x-7)^2} = -\frac{14}{(2x-7)^2}$$

$$23. f(s) = \frac{\sqrt{s}-1}{\sqrt{s}+1} = \frac{s-2\sqrt{s}+1}{s-1}$$

$$f'(s) = \frac{(1-\frac{1}{2\sqrt{s}})(s-1) - (s-2\sqrt{s}+1)}{(s-1)^2}$$

$$= \frac{s-1-\sqrt{s}+\frac{1}{2\sqrt{s}}-s+2\sqrt{s}-1}{(s-1)^2}$$

$$= \frac{\sqrt{s}+\frac{1}{2\sqrt{s}}-2}{(s-1)^2}$$

$$24. u = \frac{5\sqrt{x}-x^{-\frac{1}{2}}(5x+1)}{4x}$$

$$= \frac{10\sqrt{x}-5\sqrt{x}-x^{-\frac{1}{2}}}{4x}$$

$$= \frac{5}{4}x^{-\frac{1}{2}} - \frac{1}{4}x^{-\frac{3}{2}}$$

$$25. v = \frac{1}{3} + 1 - 4x^{-\frac{1}{2}}$$

$$v' = -x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$$

$$26. y = 2\theta^{-\frac{1}{2}} + 2\theta^{-\frac{1}{2}}$$

$$y' = -\theta^{-\frac{3}{2}} + \theta^{-\frac{3}{2}}$$

$$27. y = \frac{1}{x^4 + x^2 - x - 1}$$

$$y' = \frac{x+1 - x^4 - x^2}{x^4 + x^2 - x - 1}$$

$$28. y = \frac{x^2 + 3x + 2}{x^2 - 3x + 2}$$

$$y' = \frac{(2x+3)(x^2-3x+2) - (2x-1)(x^2+3x+2)}{(x^2-3x+2)^2}$$

$$= \frac{2x^3 - 6x^2 + 6x + 6 - 2x^3 - 6x^2 - 4x + 2x^2 + 3x + 2}{(x^2-3x+2)^2}$$

$$= \frac{-6x^2 + 12}{(x^2-3x+2)^2}$$

$$29. y = 2e^{-x} + e^{2x}$$

$$y' = -e^{-x} + e^x + 2e^{2x}$$

$$= (1+e^{-x})e^x + 2e^{2x}$$

$$30. y' = \frac{(2x+3e^x)(2e^x-1) - (2e^x-1)(x+3e^x)}{(2e^x-x)^2}$$

$$= \frac{4xe^x - 2x^2 + 6e^{2x} - 3xe^x - 2x^2e^x - 6e^{2x} + x^2 + 3e^x}{(2e^x-x)^2}$$

$$= \frac{xe^x - 2x^2e^x - x^2 + 3e^x}{(2e^x-x)^2}$$

$$31. y' = x^3e^x + 3x^2e^x$$

$$32. w' = e^{-x} + xe^{-x}$$

$$33. y' = \frac{1}{2}x^{\frac{1}{2}} - 2e^{-2x}$$

$$34. y' = -\frac{3}{5}x^{-\frac{8}{5}}$$

$$35. s = 3t^{\frac{1}{2}} +$$

$$37. w = 2^{-1.4} + \pi z^{-\frac{1}{2}}$$

$$w' = -1.4 z^{-2.4} - \frac{\pi}{2} z^{-\frac{3}{2}}$$

$$37. y = x^{\frac{2}{7}} - x^e$$

$$y' = \frac{2}{7}x^{-\frac{5}{7}} - ex^{e-1}$$

$$38. y = x^{3.2} + 2e^{1.3}$$

$$y' = 3.2x^{2.2}$$

$$39. y' = (e^{3s-1})'$$

$$= -e^{3s-2} + e^{3s-1}$$

$$= e^{3s-2}(s-1)$$

$$40. y' = e^{\theta} \left( \frac{1}{\theta^2} + \theta^{-\frac{7}{2}} \right) + e^{\theta} \left( -2\theta^{-3} - \frac{7}{2}\theta^{-\frac{9}{2}} \right)$$

$$= e^{\theta} \theta^{-2} - e^{\theta} \theta^{-\frac{7}{2}} - 2e^{\theta} \theta^{-3} - \frac{7}{2}e^{\theta} \theta^{-\frac{9}{2}}$$

$$41. y' = 2x^3 - 3x - 1$$

$$42. y' = \frac{x^4}{2x}$$

$$43. y = x^3 + 3x^2 - 5x - x^2 - 3x + 5$$

$$= x^3 + 2x^2 - 8x + 5$$

$$y' = 3x^2 - 4x - 8$$

$$44. y = 8x^3 - 6x^6 + 6x - 3x^2$$

$$y' = 24x^2 - 36x^5 + 6 - 6x$$

first derivatives

second derivatives

$$45. y = x^2 + \frac{7}{x}$$

$$y' = 2x - 7x^{-2}$$

$$y'' = 2 + 14x^{-3}$$

$$46. s = 1 + \frac{5}{t} - \frac{1}{t^2}$$

$$s' = 2t^{-3} - 5t^{-2}$$

$$s'' = 10t^{-4} - 6t^{-3}$$

$$47. r = \frac{\theta^3 + \theta^2 + \theta - \theta^2 + \theta^{-1}}{\theta^3} = 1 - \frac{1}{\theta^3}$$

$$r' = 3\theta^{-4}$$

$$r'' = -12\theta^{-5}$$

$$48. u = \frac{x^4 - x^3 + x^2 + x^2 - x^2 + x}{x^4} = 1 + \frac{1}{x^3}$$

$$u' = -3x^{-4}$$

$$u'' = 12x^{-5}$$

$$49. w = \left(\frac{1}{3z} + 1\right)(3 - z) = \frac{1}{z} - \frac{1}{3} + 3 - z = z^{-1} - z + \frac{8}{3}$$

$$w' = -z^{-2} - 1$$

$$w'' = 2z^{-3}$$

$$50. p = \frac{q^2 + 3}{q^3 - 3q^2 + 3q - 1 + q^3 + 9q^2 + 3q + 1} = \frac{q^2 + 3}{2q^3 + 16q} = \frac{q^2 + 3}{2q(q^2 + 8)} = \frac{1}{2q}$$

$$p' = -\frac{1}{2}q^{-2}$$

$$p'' = q^{-3}$$

$$51. w' = 3 \times 2z e^{2z} + 3 \cdot z^2 \cdot 2e^{2z}$$

$$= 6ze^{2z}(1+z)$$

$$w'' = 6e^{2z} + 12ze^{2z} + 12ze^{2z} + 12z^2e^{2z}$$

$$= 24ze^{2z} + 12z^2e^{2z} + 6e^{2z}$$

$$52. w = e^z(z^3 + z - z^2 - 1) = e^z(z^3 - z^2 + z - 1)$$

$$w' = e^z(3z^2 - 2z + 1) + e^z(z^3 - z^2 + z - 1)$$

$$= e^z(z^3 + 2z^2 - z)$$

$$w'' = e^z(3z^2 + 4z - 1) + e^z(z^3 + 2z^2 - z)$$

$$= e^z(z^3 + 5z^2 + 3z - 1)$$