Exercises 2.6   Answers to even problems

② (a) 2   (b) -3   (c) 1   (d) Does not exist (DNE)

(e) $+\infty$ (f) $+\infty$ (g) $+\infty$ (h) $+\infty$ (i) $-\infty$ (j) DNE

(k) 0   (l) -1

④ (a) π (b) π

⑥ (a) $\frac{1}{8}$ (b) $\frac{1}{8}$

⑧ (a) $\frac{3}{4}$ (b) $\frac{3}{4}$

⑩   0

⑫ $\frac{1}{2}$

⑭ (a) 2   (b) 2

⑯ (a) 0   (b) 0

⑱ (a) 0   (b) 0

⑳ (a) $\frac{9}{2}$ (b) $\frac{9}{2}$

㉒ (a) -1   (b) -1

㉔ $\frac{1}{2}$

㉖   0

㉘   -1

㉚ ∞

㉜ $-\frac{5}{2}$

㉝ $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+1}}{x+1} = \lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+1} \cdot \frac{1}{\sqrt{x^2}}}{(x+1) \cdot \frac{1}{\sqrt{x^2}}}$

$\left(\begin{array}{l} \text{Because } x > 0, \\ \text{we have } \sqrt{x^2} = |x| = x \end{array} \right) = \lim\limits_{x \to +\infty} \dfrac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}}$

$= \dfrac{\sqrt{1+0}}{1+0} = 1$

㉞ $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+1}}{x+1} = \lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+1} \cdot \frac{1}{\sqrt{x^2}}}{(x+1) \cdot \frac{1}{\sqrt{x^2}}}$

$\left(\begin{array}{l} \text{Because } x < 0, \\ \text{we have } \sqrt{x^2} = |x| = -x \end{array} \right)$
$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{\frac{1}{x^2}+1}}{\frac{(x+1)}{-x}}$

$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{\frac{1}{x^2}+1}}{-1-\frac{1}{x}} = -1$

㊱   3

㊳ $-\infty$

㊵ $+\infty$

㊷ $-\infty$

㊹ $+\infty$

(46) a. $+\infty$ b. $-\infty$

(48) $+\infty$

(50) $+\infty$

(52) Does not exist because $\displaystyle\lim_{\theta \to 0^{\pm}} (2 - \cot\theta) = \mp\infty$.

(54) a. $+\infty$ b. $-\infty$ c. $+\infty$ d. $-\infty$

(56) a. $+\infty$ b. $-\infty$ c. 0 d. $-\dfrac{1}{4}$

(58) a. $\dfrac{1}{8}$ b. $+\infty$ c. $+\infty$ d. 0

      e. DNE because $\displaystyle\lim_{x \to 0^{-}} f(x) = +\infty,\quad \lim_{x \to 0^{+}} f(x) = -\infty$

(60) a. $+\infty$ b. $-\infty$

(62) a. $+\infty$ b. $-\infty$ c. $-\infty$ d. $+\infty$