Math 161 Name (English) Solution

Quiz 3.8 Name (Thai) ______________

September 8, 2014 Student ID ______________

SHOW ALL WORK.

Find the derivative of the following functions.

1. (5 points) $y = \ln\sqrt{\frac{(x+1)^5}{(x+2)^{20}}}$ $= \ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)^{\frac{1}{2}} = \frac{1}{2}\ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)$

$$= \frac{1}{2}\ln(x+1)^5 - \frac{1}{2}\ln(x+2)^{20} = \frac{5}{2}\ln(x+1) - \frac{20}{2}\ln(x+2)$$

$$= \frac{5}{2}\ln(x+1) - 10\ln(x+2)$$

$$y' = \frac{5}{2}\cdot\frac{1}{x+1}\cdot(x+1)' - 10\cdot\frac{1}{x+2}\cdot(x+2)'$$

$$= \frac{5}{2}\cdot\frac{1}{x+1}\cdot(1) - 10\cdot\frac{1}{x+2}\cdot(1) = \boxed{\frac{5}{2}\cdot\frac{1}{x+1} - \frac{10}{x+2}}$$

2. (5 points) $y = (\sin x)^x$

Take ln of both sides to get $\ln y = \ln(\sin x)^x$

$$\ln y = x\cdot\ln(\sin x)$$

Differentiate both sides to get

$$\frac{1}{y}y' = x\cdot(\ln(\sin x))' + \ln(\sin x)\cdot(x)'$$

$$\frac{1}{y}y' = x\cdot\frac{1}{\sin x}\cos x + \ln(\sin x)\cdot 1$$

$$y' = y\left[\frac{x\cos x}{\sin x} + \ln(\sin x)\right]$$

$$\boxed{y' = (\sin x)^x\left[\frac{x\cos x}{\sin x} + \ln(\sin x)\right]}$$

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SHOW ALL WORK.

Find the derivative of the following functions.

1. (5 points) $y = \ln\left(\dfrac{\sqrt{x}}{1+\sqrt{x}}\right)$ Can rewrite $y = \ln\sqrt{x} - \ln(1+\sqrt{x})$

(Because $\sqrt{x} = x^{\frac{1}{2}}$) $y = \frac{1}{2}\ln x - \ln(1+\sqrt{x})$

So, $y' = \frac{1}{2}\cdot\frac{1}{x} - \frac{1}{1+\sqrt{x}}(1+\sqrt{x})'$

$= \frac{1}{2x} - \frac{1}{1+\sqrt{x}}\cdot\left(\frac{1}{2\sqrt{x}}\right)$

$= \boxed{\frac{1}{2x} - \frac{1}{2\sqrt{x}(1+\sqrt{x})}}$

2. (5 points) $y = x^{\ln x}$ Take ln of both sides to get

$\ln y = \ln x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

Differentiate both sides, we get

$\frac{1}{y}y' = (\ln x)\cdot\left(\frac{1}{x}\right) + (\ln x)\cdot\left(\frac{1}{x}\right)$

$\frac{1}{y}y' = \frac{2\ln x}{x}$

$y' = y\left(\frac{2\ln x}{x}\right)$

$= \boxed{x^{\ln x}\left(\frac{2\ln x}{x}\right)}$

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Find the derivative of the following functions.

1. (5 points) $y = \ln\left(\frac{(x^2+1)^5}{\sqrt{1-x}}\right)$

Rewrite $y = 5\ln(x^2+1) - \frac{1}{2}\ln(1-x)$

So, $y' = 5 \cdot \frac{1}{x^2+1}(x^2+1)' - \frac{1}{2} \cdot \frac{1}{1-x} \cdot (1-x)'$

$y' = 5 \cdot \frac{1}{x^2+1}(2x) - \frac{1}{2} \cdot \frac{1}{1-x} \cdot (-1)$

$y' = \frac{10x}{x^2+1} + \frac{1}{2} \cdot \frac{1}{1-x} = \boxed{\frac{10x}{x^2+1} + \frac{1}{2(1-x)}}$

2. (5 points) $y = x^{(x+1)}$ Take ln of both sides to get

~~$\ln y = (x+1)$~~

$\ln y = \ln x^{(x+1)}$

$\ln y = (x+1) \cdot \ln x$

differentiate both sides

$\frac{1}{y}y' = (x+1) \cdot (\ln x)' + (\ln x) \cdot (x+1)'$

$\frac{1}{y}y' = (x+1) \cdot \frac{1}{x} + (\ln x) \cdot (1)$

$y' = y\left[\frac{x+1}{x} + \ln x\right]$

$\boxed{y' = x^{(x+1)}\left[\frac{x+1}{x} + \ln x\right]}$

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SHOW ALL WORK.

Find the derivative of the following functions.

1. (5 points) $y = \ln\left(\frac{e^x}{1+e^x}\right)$

Can rewrite $y = \ln e^x - \ln(1+e^x)$

$y = x\ln e - \ln(1+e^x)$

$y = x - \ln(1+e^x)$

(because $\ln e = 1$)

So, $y' = 1 - \frac{1}{1+e^x}(1+e^x)'$

$= 1 - \frac{1}{1+e^x}(e^x) = \boxed{1 - \frac{e^x}{1+e^x}}$

2. (5 points) $y = x^{\sin x}$

Take ln of both sides

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

Differentiate both sides

$\frac{1}{y}y' = \sin x \cdot (\ln x)' + (\ln x) \cdot (\sin x)'$

$\frac{1}{y}y' = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$

$y' = y\left[\frac{\sin x}{x} + \ln x \cos x\right]$

$\boxed{y' = x^{\sin x}\left[\frac{\sin x}{x} + \ln x \cos x\right]}$

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SHOW ALL WORK.

Find the derivative of the following functions.

1. (5 points) $y = \ln\left(\dfrac{\sqrt{\sin x \cos x}}{1 + 2\ln x}\right) = \ln\sqrt{\sin x \cos x} - \ln(1+2\ln x)$

so, $y = \frac{1}{2}\ln\sin x + \frac{1}{2}\ln\cos x - \ln(1+2\ln x)$

$$y' = \frac{1}{2}\cdot\frac{1}{\sin x}\cdot(\sin x)' + \frac{1}{2}\cdot\frac{1}{\cos x}\cdot(\cos x)' - \frac{1}{1+2\ln x}\cdot(1+2\ln x)'$$

$$\boxed{y' = \frac{1}{2}\frac{\cos x}{\sin x} - \frac{1}{2}\frac{\sin x}{\cos x} - \frac{\frac{2}{x}}{1+2\ln x}}$$

2. (5 points) $y = (x+1)^x$

Take ln of both sides

$$\ln y = \ln(x+1)^x$$

$$\ln y = x\cdot\ln(x+1)$$

Differentiate both sides

$$\frac{1}{y}y' = x\cdot\frac{1}{x+1}\cdot 1 + \ln(x+1)\cdot 1$$

$$y' = y\left[\frac{x}{x+1} + \ln(x+1)\right]$$

$$\boxed{y' = (x+1)^x\left[\frac{x}{x+1} + \ln(x+1)\right]}$$