Midterm practice 2

1. Let $f(x) = \begin{cases} \dfrac{1}{x} & x \geq 1 \\ x^2 & x < 1 \end{cases}$

   (a) Find $\displaystyle\lim_{\Delta x \to 0^-} \frac{f(1+\Delta x) - f(1)}{\Delta x}$.

   (b) Find $\displaystyle\lim_{\Delta x \to 0^+} \frac{f(1+\Delta x) - f(1)}{\Delta x}$.

   (c) Is $f'(1)$ defined? Why or why not?

   (d) Find $f'(4)$.

2. The temperature T (in Celsius) of food in the refrigerator after t hours is given by

$$T(t) = 65 - 11t + \frac{4}{t}, \quad 1 \leq t \leq 5.$$

   Find the rate of change of the temperature with respect to time when $t = 1$.

3. Find the tangent to $y = 2e^{(1-x)}$ at $x = 1$.

4. Use linear approximation to approximate $2^{1.1}$ to three decimal places. Use $\ln 2 = 0.69$

5. The factory producing spherical containers has a machine that can measure the radius of each container with an error no greater than 0.02 centimeters. Because the weight of each container must be the same, its volume needs to be controlled. Use differential to find the maximum error of the volume of each container. The volume of sphere is given by $V = \frac{4}{3}\pi r^3$.

6. Let $f(x) = x^2 + \sin(-2x)$. Find

   (a) $f'(x)$

   (b) $f''(x)$

   (c) $\dfrac{d^3y}{dx^3}|_{x=0}$

7. Find derivatives of the following functions

   (a) $y = (x^2 + \sqrt{3x})(e^{2x})$

   (b) $y = \dfrac{\ln(3x+1)}{\cos(3x+1)}$

   (c) $y = \log_2(x^5 - 2x + 3) + \sec(2^x - 1)$

   (d) $y = (\tan(e^x + \pi))^{10}$

8. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = \dfrac{(x^2+1)^{\sin x}}{\ln(x)}$.

9. Find $\dfrac{dy}{dx}$ if $\sqrt{2x+y+1} = y\sin x$.

10. Find the following limits. If the limit does not exist, state if the limit is $+\infty$, $-\infty$, or neither. Give a reason for your answer.

    (a) $\displaystyle\lim_{x\to+\infty} \frac{x^2+3x-1}{3-2x^2}$

    (b) $\displaystyle\lim_{x\to+\infty} \frac{1+x^2}{5-3\sin x}$

    (c) $\displaystyle\lim_{x\to 0^+} (3 - \frac{1}{x} + \ln x)$

11. Evaluate $\displaystyle\lim_{x\to 0} \frac{\ln(\cos x)}{x^2}$

12. Evaluate $\displaystyle\lim_{x\to 0^+} x^2 e^{\frac{1}{x^2}}$

13. Evaluate $\displaystyle\lim_{x\to 0^+} (1-3x)^{\frac{1}{x}}$

14. Let $f(x) = \ln(2x+1)$.

    (a) Find the fourth order Maclaurin expansion of $f(x)$.

    (b) Find the Maclaurin series of $f(x)$. Write it in terms of n.

    (c) Approximate $\ln(0.2) = f(0.1)$ up to 4 decimal places using the third order Maclaurin expansion of $f(x)$.

15. Let $f(x) = 3x^5 - 5x^3 + 3$. Find all local extrema of f and where it occurs.

16. Let $f(x) = 2\sin x - \cos x$.

    (a) Use intermediate value theorem to show that there is a solution to $f(x) = 0$ in $\left[0, \frac{\pi}{2}\right]$.

    (b) Use Newton method to approximate the solution x_1 of $f(x) = 0$. Use $x_0 = \frac{\pi}{4}$.