Midterm practice 1

1. Let $f(x) = \begin{cases} 4|x-2|+1 & x < 2 \\ x^2 - 3 & x \geq 2 \end{cases}$

   (a) Find $\displaystyle\lim_{\Delta x \to 0^-} \frac{f(2+\Delta x) - f(2)}{\Delta x}$.

   (b) Find $\displaystyle\lim_{\Delta x \to 0^+} \frac{f(2+\Delta x) - f(2)}{\Delta x}$.

   (c) Is $f'(2)$ defined? Why or why not?

2. The number of cells, y, in an experiment at time t (minutes) is given by

$$y(t) = y_0(5t^2 - 3t); \quad 0 \leq t \leq 100,$$

   where y_0 is the number of cells at the beginning of the experiment. Find the rate of change of the number of cells with respect to time when $t = 40$ if there are 100 cells at the beginning of the experiment.

3. Find a point where $y = 2e^{(1-x)}$ is tangent to the line through $(0,3), (5,-2)$.

4. Find derivatives of the following functions

   (a) $y = x^2 + \sqrt{2x-1} - \cot x + 2$

   (b) $y = \dfrac{e}{x} + \log_2 x - \sec(x^2+1)$

   (c) $y = \dfrac{\pi^x}{\arccos x} + (\ln x)^2$

   (d) $y = \tan(3x^2 + x - 1) + 2^{\cos x}$

   (e) $(\ln x) \cdot (\csc 2x)$

   (f) $\sin\left(\dfrac{1}{x}\right) - e^{\left(\frac{1}{x}\right)}$

5. Let $f(x) = (2x)^{100} - x^{99} + 1$.

   (a) Find k such that the k^{th} derivative is a non-zero constant.

   (b) Find $f^{(k)}(x)$ where k is the answer from previous step.

6. Find $\dfrac{dy}{dx}$ if $\ln(x+y) = x\sin y + 7$.

7. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = \dfrac{x^{\sin x} \cdot e^{ex+e}}{\sqrt[3]{\ln(3x)}}$, $x > \dfrac{1}{3}$.

8. Use differential to approximate $\ln(1.9)$ to two decimal places. Use $\ln 2 = 0.69$

9. A semi-spherical dome has radius of 10 meters. The measurement of the radius has an error of ± 0.05 meter.

(a) Let V be volume inside the dome, r be radius. Write V in terms of r.

(b) Use differential to approximate the error from computing the volume.

10. Use Newton-Raphson (Newton's method) with $x_0 = 3$ to find x_1 which approximates $\sqrt[3]{\dfrac{78}{3}}$. Use two decimal places.

11. (a) Find the third-degree Taylor polynomial of $f(x) = \cos x$ about $x = \pi$.

    (b) Use the polynomial obtained from the previous step to approximate $\cos(3.04)$. Here, use $\pi = 3.14$.

12. Find the limits.

    (a) $\displaystyle\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x}$

    (b) $\displaystyle\lim_{x\to+\infty} \left[\ln\left(\frac{1}{x}\right) - \frac{x^3}{2x+1}\right]$

13. Evaluate the following limits.

    (a) $\displaystyle\lim_{x\to 0} \frac{7^x - 5^x}{x}$

    (b) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{1-e^{-x}} - \frac{1}{x}\right)$

    (c) $\displaystyle\lim_{x\to\frac{\pi}{2}} \sin(4x-\pi)\cdot\cot(2x-\pi)$

    (d) $\displaystyle\lim_{x\to+\infty} \left(\frac{x+e}{x+2e}\right)^x$