Homework Due Monday, November 24, 2014. **Late homework will NOT be accepted.**

1. Evaluate $\displaystyle\int \frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2}\,dx.$

2. Use trapezoidal rule with $n = 4$ to approximate $\displaystyle\int_0^2 \sqrt{1+\frac{\sin(\pi x)}{2}}\,dx.$

   Here, $\sqrt{0.5} \approx 0.707$ and $\sqrt{1.5} \approx 1.225$.

① $$\frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{Dx+E}{(x^2+1)} + \frac{Fx+G}{(x^2+1)^2}$$

$$2(x^4+1) = A(x+1)^2(x^2+1)^2 + B(x-1)(x+1)(x^2+1)^2 + (Dx+E)(x-1)(x+1)^2(x^2+1) + (Fx+G)(x-1)(x+1)^2$$

$$\Rightarrow \quad A = \frac{1}{4}, \quad B = -\frac{1}{4}, \quad C = -\frac{1}{2}, \quad D = E = 0, \quad F = 1, \quad G = -1$$

$$\Rightarrow \frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} + \frac{x-1}{(x^2+1)^2}$$

$$\int \frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2}\,dx = \int \frac{\frac{1}{4}}{x-1}dx + \int \frac{-\frac{1}{4}}{x+1}dx + \int \frac{-\frac{1}{2}}{(x+1)^2}dx + \int \frac{x-1}{(x^2+1)^2}dx$$

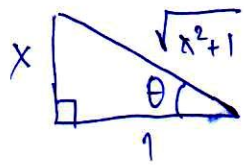
$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2}\cdot\frac{1}{(x+1)} + \int \frac{x-1}{(x^2+1)^2}dx$$

$$\int \frac{x-1}{(x^2+1)^2}dx = \int \frac{x}{(x^2+1)^2}dx - \int \frac{1}{(x^2+1)^2}dx$$

$$= \frac{1}{2}\left(-\frac{1}{x^2+1}\right) - \int \frac{1}{\sec^4\theta}\sec^2\theta\,d\theta$$

$x = \tan\theta \Rightarrow x^2+1 = \sec^2\theta$

$dx = \sec^2\theta\,d\theta$



$\sin\theta = \frac{x}{\sqrt{x^2+1}}$

$\cos\theta = \frac{1}{\sqrt{x^2+1}}$

$$= \int \cos^2\theta\,d\theta$$

$$= \int \frac{1}{2}d\theta + \int \frac{1}{2}\cos 2\theta\,d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$

$$= \frac{1}{2}\tan^{-1}x + \frac{1}{2}\frac{x}{\sqrt{x^2+1}} + C$$

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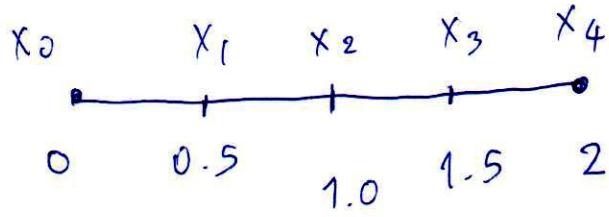

$$\Delta x = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

$$y_0 = \sqrt{1 + \frac{\sin(0)}{2}} = \sqrt{1} = 1$$

$$y_1 = \sqrt{1 + \frac{\sin\frac{\pi}{2}}{2}} = \sqrt{1.5} \approx 1.225$$

$$y_2 = \sqrt{1 + \frac{\sin\pi}{2}} = \sqrt{1} = 1$$

$$y_3 = \sqrt{1 + \frac{\sin\frac{3\pi}{2}}{2}} = \sqrt{0.5} \approx 0.707$$

$$y_4 = \sqrt{1 + \frac{\sin 2\pi}{2}} = \sqrt{1} = 1$$

$$T = \frac{\Delta x}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$= \frac{0.5}{2}\left[1 + 2(1.225) + 2(1) + 2(0.707) + 1\right]$$

$$= 1.966$$