Homework Due Monday, November 20, 2014. **Late homework will NOT be accepted.**

1. Find the area of the region in the first quadrant that is enclosed by the coordinate axes and the curve $y = (2 - x^2)^{3/2}$.

2. Evaluate $\displaystyle\int \frac{2x+1}{x^2 - 3x + 2}\, dx$.

$$\frac{1}{x+a} + \frac{1}{x+b} = \frac{(x+b) + (x+a)}{(x+a)(x+b)}$$

$$= \frac{2x+a+b}{(x+a)(x+b)}$$

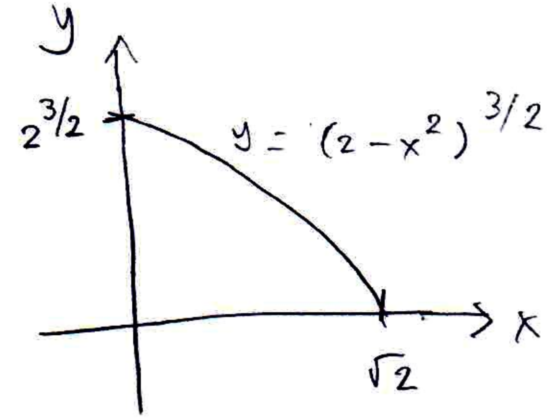
In general, if you see

$$\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{x+2}$$

then solve for A, B

$$1 = A(x+2) + B(x+1)$$

Then plug-in $x=-2$, $x=-1$ to solve for A, B.

① 

$$\text{Area} = \int_0^{\sqrt{2}} (2-x^2)^{3/2}\,dx$$

Consider $\int (2-x^2)^{3/2}\,dx$

Let $x = \sqrt{2}\sin\theta \Rightarrow dx = \sqrt{2}\cos\theta\,d\theta$

$$(2-x^2)^{3/2} = (2-2\sin^2\theta)^{3/2}$$
$$= 2^{3/2}(1-\sin^2\theta)^{3/2}$$
$$= 2^{3/2}\cos^3\theta$$

when $x = 0 \Rightarrow 0 = \sqrt{2}\sin\theta \Rightarrow \theta = 0$

when $x = \sqrt{2} \Rightarrow \sqrt{2} = \sqrt{2}\sin\theta \Rightarrow \sin\theta = 1$
$\Rightarrow \theta = \frac{\pi}{2}$

so, $\int (2-x^2)^{3/2} = \int 2^{3/2}\cos^3\theta\,\sqrt{2}\cos\theta\,d\theta$

$$= 4\int \cos^4\theta\,d\theta$$
$$= 4\int \left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$
$$= \int 1 + 2\cos 2\theta + \cos^2 2\theta\;d\theta$$
$$= \theta + \sin 2\theta + \int \cos^2 2\theta\,d\theta$$
$$= \theta + \sin 2\theta + \frac{1}{2}\int (1+\cos 4\theta)\,d\theta$$
$$= \theta + \sin 2\theta + \frac{1}{2}\theta + \frac{1}{8}\sin 4\theta + C$$
$$= \frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta + C$$

Therefore, area $= \int_0^{\sqrt{2}} (2-x^2)^{3/2}\,dx$

$$= \left[\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta\right]_0^{\frac{\pi}{2}}$$
$$= \frac{3}{2}\left[\frac{\pi}{2} - 0\right] + [\sin\pi - \sin 0] + \frac{1}{8}[\sin 2\pi - \sin 0]$$
$$= \frac{3\pi}{4}$$

---

② $$\int \frac{2x+1}{x^2-3x+2}\,dx = \int \frac{2x+1}{(x-2)(x-1)}\,dx$$

$$\frac{2x+1}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$$

$$\frac{2x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$2x+1 = A(x-2) + B(x-1)$$

Plug-in $x = 1 \Rightarrow \quad 2+1 = A(1-2) \quad \Rightarrow A = -3$

Plug-in $x = 2 \Rightarrow \quad 4+1 = B(2-1) \quad \Rightarrow B = 5$

So, $\dfrac{2x+1}{(x-1)(x-2)} = \dfrac{-3}{x-1} + \dfrac{5}{x-2}$

$$\int \frac{2x+1}{x^2-3x+2}\,dx = \int \frac{-3}{x-1}\,dx + \int \frac{5}{x-2}\,dx$$

$$= -3\ln|x-1| + 5\ln|x-2| + C$$