Homework Due Thursday, Novemver 13, 2014. **Late homework will NOT be accepted.**

1. Evaluate $\int x^2 e^{-3x}\, dx$.

2. Use the cylindrical shell method to find the volume of the solid generated by revolving the region in the first quadrant enclosed by $y = \log_2 x, x \in [1, 2]$, about the line $x = \frac{1}{2}$.

3. Evaluate $\int_0^{\pi/2} \sin^7\left(\frac{x}{2}\right) \cos^5\left(\frac{x}{2}\right)\, dx$.

① Use integration by parts twice

$$\int x^2 e^{-3x}\,dx = -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}\int x e^{-3x}\,dx$$

$$\boxed{\begin{array}{ll} u = x^2 & dv = e^{-3x}\,dx \\ du = 2x\,dx & v = -\frac{1}{3}e^{-3x} \end{array}}$$

$$\boxed{\begin{array}{ll} u = x & dv = e^{-3x}\,dx \\ du = dx & v = -\frac{1}{3}e^{-3x} \end{array}}$$

$$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}\left[-\frac{1}{3}x e^{-3x} + \frac{1}{3}\int e^{-3x}\,dx\right]$$

$$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} + \frac{2}{9}\int e^{-3x}\,dx$$

$$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} + \frac{2}{9}\left(-\frac{1}{3}\right)e^{-3x} + C$$

$$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} - \frac{2}{27}e^{-3x} + C$$

Use table

x^2	+	e^{-3x}
$2x$	−	$-\frac{1}{3}e^{-3x}$
2	+	$\frac{1}{9}e^{-3x}$
0		$-\frac{1}{27}e^{-3x}$

$$\int x^2 e^{-3x}\,dx = -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} - \frac{2}{27}e^{-3x} + C$$

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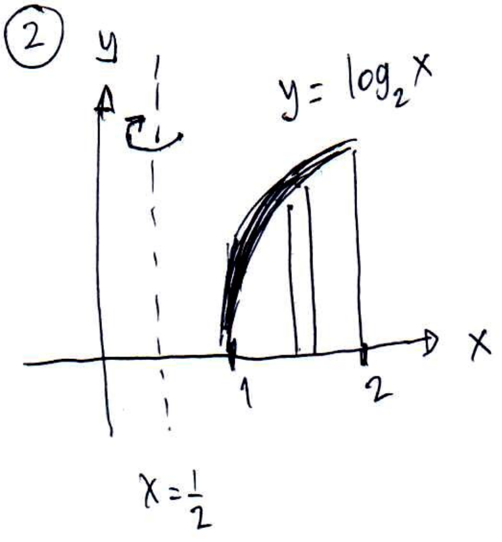
②



$$V = \int_1^2 2\pi r h\,dx$$

~~u = x d~~

$$\begin{array}{ll} u = \log_2 x & dv = x \\ du = \frac{1}{x\ln 2}\,dx & v = \frac{1}{2}x^2 \end{array}$$

$$= \int_1^2 2\pi x \log_2 x\,dx$$

$$= 2\pi\int_1^2 x\log_2 x\,dx$$

$$= 2\pi\left[\frac{1}{2}x^2\log_2 x\right]_1^2 - 2\pi\left[\int_1^2 \frac{1}{2\ln 2}x\,dx\right]$$

$$= 2\pi\left[\,2-0\,\right] - 2\pi \cdot \frac{1}{2\ln 2}\int_1^2 x\,dx$$

$$= 4\pi - \frac{\pi}{\ln 2}\left[\frac{1}{2}x^2\right]_1^2 = 4\pi - \frac{\pi}{\ln 2}\left[\frac{1}{2}\cdot 2^2 - \frac{1}{2}\cdot 1^2\right]$$

$$= 4\pi - \frac{\pi}{\ln 2}\left[2-\frac{1}{2}\right] = 4\pi - \frac{3\pi}{2\ln 2}$$

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③ consider $\int \sin^7\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)dx$

$$= \int \sin^7\left(\frac{x}{2}\right)\cos^4\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx = \int \sin^7\left(\frac{x}{2}\right)\left(\cos^2\left(\frac{x}{2}\right)\right)^2\cos\left(\frac{x}{2}\right)dx$$

$$\boxed{\begin{aligned} &\text{let } u = \sin\left(\frac{x}{2}\right) \\ &du = \cos\left(\frac{x}{2}\right)\cdot\frac{1}{2}dx \\ &2du = \cos\left(\frac{x}{2}\right)dx \end{aligned}}$$

$$= \int \sin^7\left(\frac{x}{2}\right)\left[1-\sin^2\left(\frac{x}{2}\right)\right]^2\cos\left(\frac{x}{2}\right)dx$$

$$= \int u^7\left[1-u^2\right]^2 \cdot 2du$$

$$= 2\int u^7\left(1-2u^2+u^4\right)du$$

$$= 2\int\left(u^7-2u^9+u^{11}\right)du$$

$$= 2\left[\frac{1}{8}u^8 - \frac{2}{10}u^{10} + \frac{1}{12}u^{12}\right]$$

$$= 2\left[\frac{1}{8}\sin^8\left(\frac{x}{2}\right) - \frac{1}{5}\sin^{10}\left(\frac{x}{2}\right) + \frac{1}{12}\sin^{12}\left(\frac{x}{2}\right)\right]$$

so, $\int_0^{\pi/2}\sin^7\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)dx = \left[\frac{1}{4}\sin^8\left(\frac{x}{2}\right) - \frac{2}{5}\sin^{10}\left(\frac{x}{2}\right) + \frac{1}{6}\sin^{12}\left(\frac{x}{2}\right)\right]_0^{\pi/2}$

$$= \frac{1}{4}\cdot\frac{1}{16} - \frac{2}{5}\cdot\frac{1}{32} + \frac{1}{6}\cdot\frac{1}{64}$$

$$= \frac{11}{1920}$$