Homework Due Monday, October 20, 2014.

1. Evaluate $\int \sin^5\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\,d\theta$.

2. Evaluate $\int x(x-1)^{10}\,dx$.

3. Evaluate $\int \sqrt{\frac{2x-1}{x^5}}\,dx$.

4. If we haven't done this in class, approximate the area of the region R from Example 4.

① Let $u = \sin\frac{\theta}{2}$

so, $\frac{du}{d\theta} = \cos\frac{\theta}{2} \cdot \frac{1}{2}$

$\frac{2\,du}{\cos\frac{\theta}{2}} = d\theta$

$\int \sin^5\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) d\theta$

$= \int u^5 \cdot \cos\left(\frac{\theta}{2}\right) \cdot \frac{2\,du}{\cos\left(\frac{\theta}{2}\right)}$

$= 2\int u^5\,du$

$= 2 \cdot \frac{u^6}{6} + C = \frac{1}{3}\sin^6\frac{\theta}{2} + C$

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② Let $u = x-1$

$(u+1 = x)$

so, $\frac{du}{dx} = 1$

$du = dx$

$\int x(x-1)^{10}\,dx = \int (u+1)u^{10}\,du$

$= \int u^{11} + u^{10}\,du$

$= \frac{1}{12}u^{12} + \frac{1}{11}u^{11} + C$

$= \frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + C$

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③ $\int \sqrt{\frac{2x-1}{x^5}}\,dx = \int \sqrt{\frac{2x-1}{x^4 \cdot x}}\,dx = \int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx$

Let $u = 2 - \frac{1}{x}$

so $\frac{du}{dx} = \frac{1}{x^2}$

$x^2\,du = dx$

$= \int \frac{1}{x^2}\sqrt{u} \cdot x^2\,du = \int \sqrt{u}\,du$

$= \frac{2}{3}u^{3/2} + C = \frac{2}{3}\left(2-\frac{1}{x}\right)^{3/2} + C$

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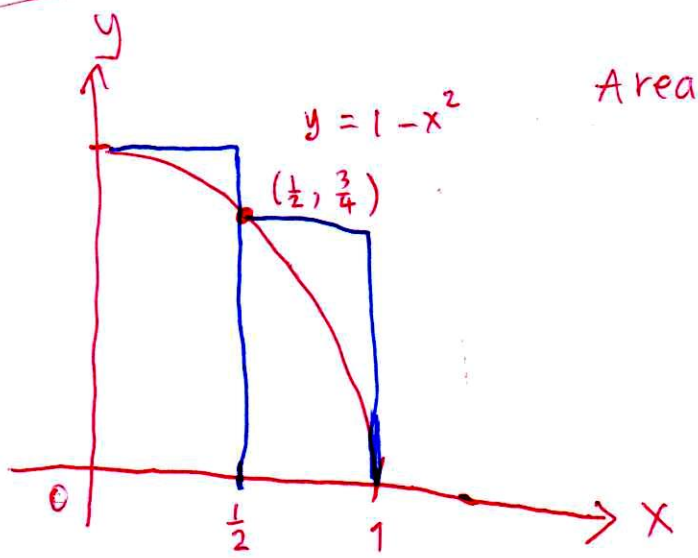
④



Area $\approx$ □ + □

$= (\frac{1}{2})(1) + (\frac{1}{2})(\frac{3}{4})$

$= \frac{1}{2} + \frac{3}{8}$

$= \frac{7}{8}$