Homework (Due Thursday, October 30, 2014.) Read the handout and do the following problems.

1. Let $f(x) = -x^2 + 4$. Find c such that $f(c)$ is the average value of f on $[-2, 2]$.

2. Find area between the graph of f and the $x-$axis.

   (a) $f(x) = |x - 1|, \qquad 0 \le x \le 2$

   (b) $f(x) = x(x - 1)(x + 3)$

3. Find the derivatives of the following functions.

   (a) $F(x) = \displaystyle\int_0^{e^x} \sqrt{t^2 + 1}\, dt$

   (b) $G(x) = \displaystyle\int_x^{e^x} \sqrt{t^2 + 1}\, dt$

   (c) $H(x) = \displaystyle\int_1^{x} x\sqrt{t^2 + 1}\, dt$

   Hint: Rewrite H as a product of two functions:

   $$H(x) = x \cdot \int_1^{x} \sqrt{t^2 + 1}\, dt$$

(1) $f(x) = -x^2+4, \quad x \in [-2,2]$

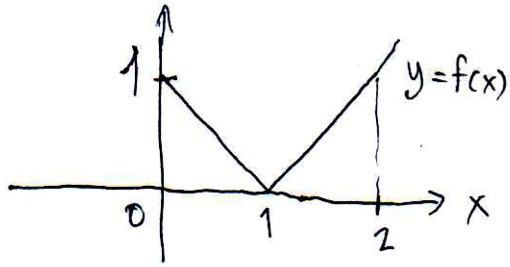
$$AV(f) = \frac{1}{2-(-2)} \int_{-2}^{2} (-x^2+4)\,dx = \frac{1}{4}\int_{-2}^{2}(-x^2+4)\,dx = \frac{1}{4}\left[-\frac{1}{3}x^3+4x\right]_{-2}^{2}$$

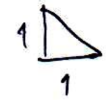
$$= \frac{1}{4}\left[\left(-\frac{1}{3}\cdot 8 + 4\cdot 2\right) - \left(-\frac{1}{3}\cdot(-8) + 4(-2)\right)\right] = \frac{8}{3}$$

$$f(c) = \frac{8}{3} \Rightarrow -c^2+4 = \frac{8}{3} \Rightarrow 4-\frac{8}{3} = c^2$$

$$\boxed{c = \frac{\pm 2}{\sqrt{3}}}$$

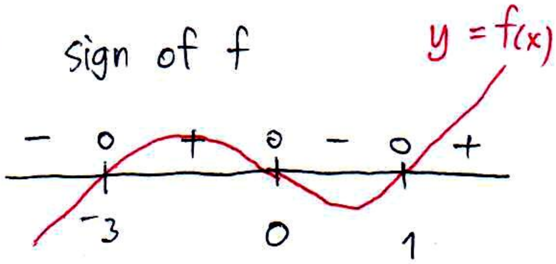
(2) (a)



Area =  + 

$$= \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = \boxed{1}$$

(b) sign of f



$$\text{Total area} = \int_{-3}^{0} f(x)\,dx + \left|\int_0^1 f(x)\,dx\right|$$

$$= \int_{-3}^{0} x(x-1)(x+3)\,dx + \left|\int_0^1 x(x-1)(x+3)\,dx\right|$$

$$= \frac{45}{4} + \left|-\frac{7}{12}\right| = \frac{45}{4} + \frac{7}{12} = \boxed{\frac{71}{6}}$$

(3) (a) $F'(x) = \sqrt{(e^x)^2+1}\cdot e^x$ (chain Rule)

(b) $G(x) = -\int_a^x \sqrt{t^2+1}\,dt + \int_a^{e^x} \sqrt{t^2+1}\,dt$

$G'(x) = -\sqrt{x^2+1} + \sqrt{(e^x)^2+1}\cdot e^x$

(c) $H'(x) = x\cdot\sqrt{x^2+1} + \left(\int_1^x \sqrt{t^2+1}\,dt\right)\cdot 1$

(product rule)