Homework (Due Monday, October 20, 2014.) Read the handout and do the following problems.

1. Use a formula from geometry to evaluate $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$.

2. Let $f(x) = x^2$. Use an upper sum with four rectangle with equal base to approximate the area between the graph of f and the $x-$axis from $x = 0$ to $x = 2$.

3. Write the following sums without sigma notation. Then evaluate them.

   Example: $\sum_{k=1}^{4} (-1)^{k+1} k = 1 - 2 + 3 - 4 = -2.$

   (a) $\sum_{k=3}^{6} \frac{k}{k+1}$

   (b) $\sum_{k=1}^{4} \sin \frac{\pi}{k}$

4. Express the following sums in terms of sigma notation.

   Example: $1 + 3 + 5 + 7 + 9 = \sum_{k=1}^{5} 2k - 1.$

   (a) $-1 + 2 - 4 + 8 - 16 + 32$

   (b) $1 + 10 + 100 + 1000$

5. Use formulas (1)-(3) to evaluate the following sums.

Example:

$$\begin{aligned}\sum_{k=1}^{100} 2k+1 &= 2\sum_{k=1}^{100} k + \sum_{k=1}^{100} 1 \\ &= 2\cdot\frac{(100)(101)}{2} + 100\cdot 1 = 10200.\end{aligned}$$

(a) $\sum_{k=1}^{20} 3 - k^3$

(b) $\sum_{k=11}^{30} -2k(k+1)$

6. Write the Limits of Riemann sums into definite integrals.

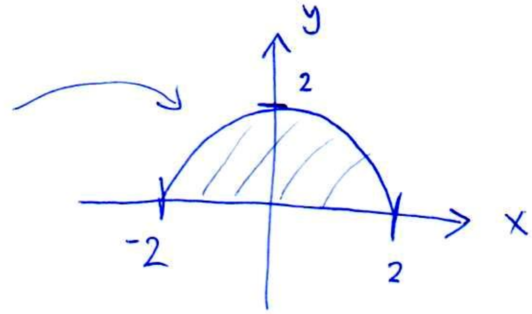
(a) $\lim_{\|P\|\to 0} \sum_{k=1}^{n} (3c_k^2 - 2c_k + 5)\Delta x_k$ where P is a partition of $[-1,3]$.

(b) $\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \sin(c_k^2)\Delta x_k$ on $[0, 2\pi]$.

7. Follow the same steps as in example 4 to find the area $\int_0^2 x^2\,dx$.

8. Use the Fundamental theorem of Calculus (Pt. 2) to confirm your answer from problem 7.

① $\displaystyle\int_{-2}^{2} \sqrt{4-x^2}\,dx = \text{Area}$

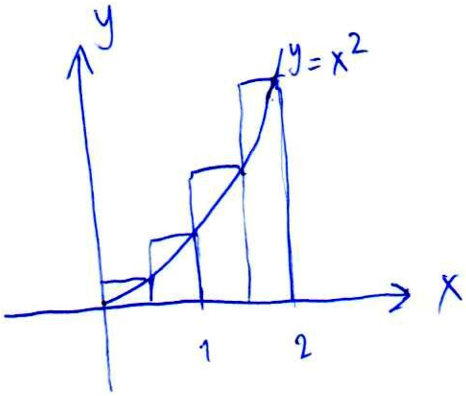


$= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(2^2)$

$= 2\pi$

②



$\Delta x_i = \frac{1}{2}, \quad i = 1, \ldots, 4$

$c_1 = \frac{1}{2} \qquad f(c_1) = \frac{1}{4}$

$c_2 = 1 \qquad f(c_2) = 1$

$c_3 = \frac{3}{2} \qquad f(c_3) = \frac{9}{4}$

$c_4 = 2 \qquad f(c_4) = 4$

Upper sum

$= \displaystyle\sum_{i=1}^{4} f(c_i)\,\Delta x_i$

$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}\right) + \left(\frac{9}{4}\right)\left(\frac{1}{2}\right) + (4)\left(\frac{1}{2}\right)$

$= \frac{1}{8} + \frac{1}{2} + \frac{9}{8} + 2$

$= \frac{15}{4} = 3.75$

③ (a) $\displaystyle\sum_{k=3}^{6} \frac{k}{k+1} = \frac{3}{3+1} + \frac{4}{4+1} + \frac{5}{5+1} + \frac{6}{6+1} = \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} = \frac{1361}{420}$

(b) $\displaystyle\sum_{k=1}^{4} \sin\frac{\pi}{4} = \sin\pi + \sin\frac{\pi}{2} + \sin\frac{\pi}{3} + \sin\frac{\pi}{4}$

$= 0 + 1 + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} = \frac{2+\sqrt{3}+\sqrt{2}}{2}$

④ (a) $\displaystyle\sum_{i=1}^{6} (-1)^i\, 2^{i-1}$

(b) $\displaystyle\sum_{i=1}^{4} 10^{i-1}$

(5) (a) $\sum_{k=1}^{20} 3 - k^3 = \sum_{k=1}^{20} 3 - \sum_{k=1}^{20} k^3 = 3(20) - \left[\frac{20(20+1)}{2}\right]^2$

$= 60 - 210^2 = 60 - 44100 = -44040$

(b) $\sum_{k=11}^{30} -2k(k+1) = \sum_{k=11}^{30}(-2k^2-2k) = \cancel{\sum_{k=1}^{30}(-2k^2-2k) - \sum_{k=1}^{10}}$

$= \sum_{k=1}^{30}(-2k^2-2k) - \sum_{k=1}^{10}(-2k^2-2k)$

$= -2\left[\frac{(30)(30+1)(60+1)}{6}\right] - 2\left[\frac{30(30+1)}{2}\right]$

$+ 2\left[\frac{(10)(10+1)(20+1)}{6}\right] + 2\left[\frac{10(10+1)}{2}\right]$

$= -2(9455) - 2(465) + 2(385) + 2(55)$

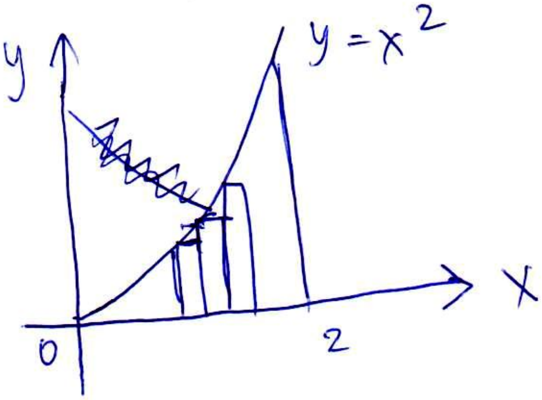
$= -18910 - 930 + 770 + 110$

$= -18960$

(6) (a) $\int_{-1}^{3}(3x^2-2x+5)\,dx$

(b) $\int_{0}^{2\pi} \sin(x^2)\,dx$

(7)



$\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$c_k = 0 + (k-1)\Delta x$

$= \frac{2(k-1)}{n}$

$f(c_k) = \left[\frac{2(k-1)}{n}\right]^2 \Rightarrow A_k = \frac{2}{n} \times \left[\frac{2(k-1)}{n}\right]^2 = \frac{8}{n^3}(k-1)^2$

so, $S_n = \sum_{k=1}^{n} A_k = \sum_{k=1}^{n} \frac{8}{n^3}(k-1)^2 = \frac{8}{n^3}\sum_{k=1}^{n}(k-1)^2$

$= \frac{8}{n^3}\sum_{k=1}^{n}(k^2-2k+1) = \frac{8}{n^3}\left[\sum_{k=1}^{n} k^2 - 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1\right]$

$= \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - 2\,\frac{n(n+1)}{2} + n\right]$

$\Rightarrow \int_0^2 x^2 dx = \lim_{n\to\infty} S_n = 8\left[\frac{2}{6} - 0 + 0\right] = \frac{8}{3}$

(8) $\int_0^2 x^2 dx = \frac{1}{3}x^3\Big|_0^2 = \frac{1}{3}2^3 - \frac{1}{3}0^3 = \frac{8}{3}$.