HW 4.5 Find limits using L'Hôpital's Rule

① $\lim_{\theta \to 0} \frac{\sin 5\theta}{\tan 2\theta}$ $\left(\frac{0}{0} \text{ form}\right)$

$$\text{L.R.} = \lim_{\theta \to 0} \frac{(\sin 5\theta)'}{(\tan 2\theta)'} = \lim_{\theta \to 0} \frac{5\cos 5\theta}{2\sec^2 2\theta} = \frac{5\cos 0}{2\sec^2 0}$$

$$= \frac{5}{2}$$

② $\lim_{x \to 0^+} x \cdot (\ln x)^2$ $\left(0 \cdot (-\infty)^2 = 0 \cdot \infty \text{ form}\right)$

$$= \lim_{x \to 0^+} \frac{(\ln x)^2}{\frac{1}{x}} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

L.R.

$$= \lim_{x \to 0^+} \frac{2\ln x \cdot \frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} \frac{2\ln x}{-\frac{1}{x}} \quad \left(\frac{-\infty}{-\infty} \text{ form}\right)$$

L.R.

$$= \lim_{x \to 0^+} \frac{2 \cdot \frac{1}{x}}{\frac{1}{x^2}} = \lim_{x \to 0^+} 2x = 2 \cdot 0 = 0$$