HW 3 solution

(1) Find y' if

(a) $2xy + y^2 = x + y$

(b) $y^2 = \dfrac{x-1}{x+1}$

(2) $y^4 = y^2 - x^2$ at point $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$.

Solution

1 (a), $(2xy)' + (y^2)' = (x+y)'$

$$(2x)y' + (2x)'y + 2y(y') = 1 + y'$$

$$2xy' + 2y + 2yy' = 1 + y'$$

$$2xy' + 2yy' - y' = 1 - 2y$$

$$(2x + 2y - 1)y' = 1 - 2y$$

$$y' = \frac{1-2y}{2x+2y-1}$$

2(a) $(y^2)' = \left(\frac{x-1}{x+1}\right)'$

$$2yy' = \frac{(x+1)(x-1)' - (x-1)(x+1)'}{(x+1)^2}$$

$$2yy' = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$y' = \frac{1}{2y} \cdot \frac{2}{(x+1)^2}$$

$$y' = \frac{1}{y(x+1)^2}$$

2. $(y^4)' = (y^2 - x^2)'$

$$4y^3 y' = 2yy' - 2x$$

$$(4y^3 - 2y)y' = -2x$$

$$y' = \frac{-2x}{4y^3 - 2y}$$

so, slope at $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$ is given by

$$y' = \frac{-2\left(\frac{\sqrt{3}}{4}\right)}{4\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)} = \frac{-\frac{\sqrt{3}}{2}}{4\left(\frac{1}{8}\right) - 2\left(\frac{1}{2}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2} - 1}$$

$$= \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$