HW 1

① Find limits

(a) $\lim_{x \to 5} \dfrac{x-5}{x^2-25}$

Solution $\lim_{x \to 5} \dfrac{x-5}{x^2-25} = \lim_{x \to 5} \dfrac{x-5}{(x-5)(x+5)} = \lim_{x \to 5} \dfrac{1}{x+5} = \dfrac{1}{10}$

(b) $\lim_{x \to 2} \dfrac{\sqrt{x^2+12}-4}{x-2}$

Solution $\lim_{x \to 2} \dfrac{\sqrt{x^2+12}-4}{x-2} = \lim_{x \to 2} \dfrac{\sqrt{x^2+12}-4}{x-2} \cdot \dfrac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}$

$= \lim_{x \to 2} \dfrac{(x^2+12)-16}{(x-2)(\sqrt{x^2+12}+4)} = \lim_{x \to 2} \dfrac{x^2-4}{(x-2)(\sqrt{x^2+12}+4)}$

$= \lim_{x \to 2} \dfrac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+12}+4)} = \lim_{x \to 2} \dfrac{x+2}{\sqrt{x^2+12}+4}$

$= \dfrac{2+2}{\sqrt{2^2+12}+4} = \dfrac{1}{2}$

② Let $f(x) = \begin{cases} 3-x, & x<2 \\ 2, & x=2 \\ \frac{x}{2}, & x>2 \end{cases}$ check if f is continuous at $x=2$.

Ansewer f is not continuous at $x=2$.

Reason $\left.\begin{array}{l} \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3-x) = 3-2 = 1 \\ \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x}{2} = \frac{2}{2} = 1 \end{array}\right\}$ so, $\lim_{x \to 2} f(x) = 1$

However, $f(2) = 2 \neq \lim_{x \to 2} f(x)$,