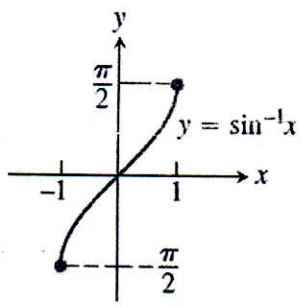
Inverse Trigonometric Functions

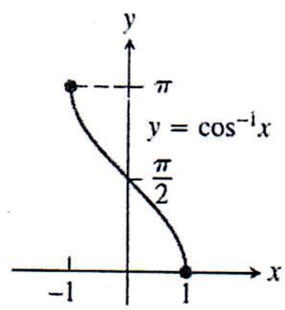
Trig.	domain	range
$\sin x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le y \le 1$
$\cos x$	$0 \le x \le \pi$	$-1 \le y \le 1$
$\tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$-\infty < y < \infty$
$\cot x$	$0 < x < \pi$	$-\infty < y < \infty$
$\sec x$	$0 \le x \le \pi,\ x \ne \frac{\pi}{2}$	$y \le -1$ or $y \ge 1$
$\csc x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2},\ x \ne 0$	$y \le -1$ or $y \ge 1$

Inverse Trig.	domain	range	derivative
$\sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$	$-\frac{1}{1+x^2}$
$\sec^{-1} x$	$x \le -1$ or $x \ge 1$	$0 \le y \le \pi,\ y \ne \frac{\pi}{2}$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\csc^{-1} x$	$x \le -1$ or $x \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2},\ y \ne 0$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$

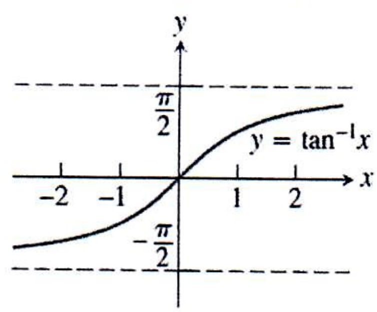
Domain: $-1 \le x \le 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



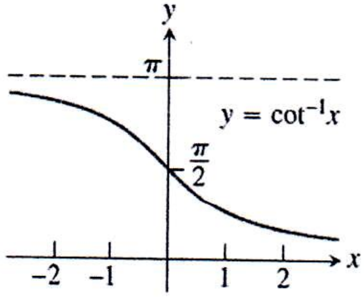
Domain: $-1 \le x \le 1$
Range: $0 \le y \le \pi$



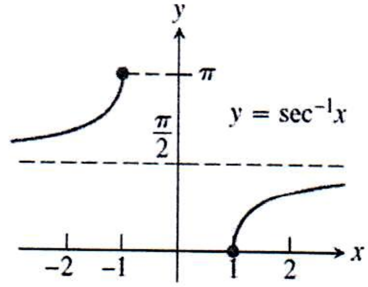
Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



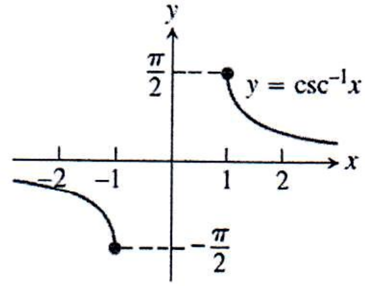
Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$



Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi,\ y \ne \frac{\pi}{2}$



Domain: $x \le -1$ or $x \ge 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2},\ y \ne 0$



Chain rules

$$\frac{d}{dx}\sin^{-1}[u(x)] = \frac{1}{\sqrt{1-[u(x)]^2}}u'(x), \quad |u(x)| < 1$$

$$\frac{d}{dx}\cos^{-1}[u(x)] = -\frac{1}{\sqrt{1-[u(x)]^2}}u'(x), \quad |u(x)| < 1$$

$$\frac{d}{dx}\tan^{-1}[u(x)] = \frac{1}{1+[u(x)]^2}u'(x)$$

$$\frac{d}{dx}\cot^{-1}[u(x)] = -\frac{1}{1+[u(x)]^2}u'(x)$$

$$\frac{d}{dx}\sec^{-1}[u(x)] = \frac{1}{|u(x)|\sqrt{[u(x)]^2-1}}u'(x), \quad |u(x)| > 1$$

$$\frac{d}{dx}\csc^{-1}[u(x)] = -\frac{1}{|u(x)|\sqrt{[u(x)]^2-1}}u'(x), \quad |u(x)| > 1$$

Related rates

1. A spherical balloon is being inflated so that its spherical shape is maintained at all time.
   (a) Find the relation between the volume of the air inside the balloon (V) and its radius (r).
   (b) If the air is being pumped into the balloon at a constant rate (say 1 cm^3 per second), will the radius of the balloon increase at a constant rate?
   (c) On the other hand, if we inflate the balloon so that its radius is expanding at a constant rate (say 2 cm per second), does that mean we have to pump in the air at a constant rate?

2. (Balloon problem revisited) A spherical balloon is being inflated at the rate of 1 cm^3 per second so that its spherical shape is maintained at all time. Find the rate at which its radius (r) is increasing when $r = 3$ cm.

   Related Rates Problem Strategy

   (1) Draw a picture and name the variables and constants. Use t for time. Express rate as derivative.
   (2) Write down the known information (in terms of the symbols you have chosen).
   (3) Write down the rate you are asked to find. At what instance?
   (4) Write an equation that relates the variables whose rates are involved. Eliminate other variables.
   (5) Differentiate with respect to t.
   (6) Plug-in known values to find the unknown rate.