① 1.1 $\int \left(e^{\pi x} + \frac{4}{\sqrt{3-x^2}} + \frac{\ln 2}{x^2}\right) dx$

$= \frac{1}{\pi} e^{\pi x} + 4\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + \ln 2\,(-x^{-1}) + C$

1.2 Let $u = 5 - e^{2x+1}$ $\Rightarrow \frac{du}{dx} = -e^{2x+1} \cdot 2 \Rightarrow \frac{du}{-2e^{2x+1}} = dx$

$\int e^{2x}(5-e^{2x+1})^{10} dx = \int e^{2x}(u)^{10} \frac{du}{-2e^{2x+1}} = \int e^{2x} u^{10} \frac{du}{2e^{2x} \cdot e^{1}}$

$= \frac{-1}{2e} \int u^{10} du = \frac{-1}{2e} \cdot \frac{1}{11} u^{11} + C = \frac{-1}{22e}(5 - e^{2x+1})^{11} + C$

1.3 $\int \frac{2^{(10+\ln x)}}{x} dx = \int \frac{2^u}{x}(x\,du) = \int 2^u du = \frac{2^u}{\ln 2} + C$

let $u = 10 + \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$x\,du = dx$

$= \frac{2^{10+\ln x}}{\ln 2} + C$

1.4 $\int \frac{\operatorname{cosec}(\sqrt{x})\cot(\sqrt{x})}{\sqrt{x}} dx$

let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x}\,du = dx$

$= \int \frac{\operatorname{cosec}(u)\cot(u)}{\sqrt{x}} 2\sqrt{x}\,du$

$= 2\int \operatorname{cosec}(u)\cot(u)\,du = -2\operatorname{cosec}(u) + C$

$= -2\operatorname{cosec}(\sqrt{x}) + C$

1.5 $\int \frac{\sec^2(2x)}{1+\tan(2x)} dx = \int \frac{1}{u} \cdot \frac{1}{2} du = \frac{1}{2}\ln|u| + C$

let $u = 1 + \tan(2x)$

$\frac{du}{dx} = \sec^2(2x) \cdot 2$

$\frac{1}{2} du = \sec^2(2x)\,dx$

$= \frac{1}{2}\ln|1 + \tan(2x)| + C$

1.6 $\int \frac{x}{9+x^4} dx = \int \frac{x}{9+(x^2)^2} dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2} du = x\,dx$

$= \int \frac{1}{9+u^2} \cdot \frac{1}{2} du = \frac{1}{2} \cdot \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + C = \frac{1}{6}\tan^{-1}\left(\frac{x^2}{3}\right) + C$

(2) $\int \sin(\ln x)\,dx = x\sin(\ln x) - \int \cos(\ln x)\,dx$

$u = \sin(\ln x) \qquad dv = dx$

$du = \cos(\ln x)\cdot\frac{1}{x}\,dx \qquad v = x$

$= x\sin(\ln x) - \left[x\cos(\ln x) + \int \sin(\ln x)\,dx\right]$

$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$

$u = \cos(\ln x) \qquad dv = dx$

$du = -\sin(\ln x)\cdot\frac{1}{x}\,dx \qquad v = x$

$\Rightarrow \quad 2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x)$

$\int \sin(\ln x)\,dx = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$

(3) $\int \sin^3(2x+1)\cos^2(2x+1)\,dx$

let $u = \cos(2x+1)$

$\frac{du}{dx} = -\sin(2x+1)\cdot 2$

$-\frac{1}{2}du = \sin(2x+1)\,dx$

$= \int \sin(2x+1)\sin^2(2x+1)\cos^2(2x+1)\,dx$

$= \int \sin(2x+1)\,(1-\cos^2(2x+1))\cos^2(2x+1)\,dx$

$= -\frac{1}{2}\int (1-u^2)u^2\,du = -\frac{1}{2}\int u^2 - u^4\,du$

$= -\frac{1}{2}\left[\frac{1}{3}u^3 - \frac{1}{5}u^5\right] + C$

$= -\frac{1}{6}u^3 + \frac{1}{10}u^5 + C = -\frac{1}{6}\cos^3(2x+1) + \frac{1}{10}\cos^5(2x+1) + C$

(4) $\int (-5)\cos(15x)\cos(5x)\,dx = -\frac{5}{2}\int \cos 20x + \cos 10x\,dx$

$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$= -\frac{5}{2}\left[\frac{1}{20}\sin(20x) + \frac{1}{10}\sin(10x)\right] + C$

(5) $\int \frac{1}{(1-x^2)^{5/2}}\,dx = \int \frac{1}{(\cos^2\theta)^{5/2}}\cos\theta\,d\theta = \int \frac{1}{\cos^4\theta}\,d\theta = \int \sec^4\theta\,d\theta$

$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$1 - x^2 = 1 - \sin^2\theta = \cos^2\theta$

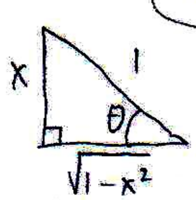
$= \int \sec^2\theta\cdot\sec^2\theta\,d\theta = \int (\tan^2\theta + 1)\sec^2\theta\,d\theta$

$u = \tan\theta \Rightarrow du = \sec^2\theta\,d\theta$

$= \int (u^2+1)\,du = \frac{1}{3}u^3 + u + C$

$= \frac{1}{3}\tan^3\theta + \tan\theta + C$

$= \frac{1}{3}\frac{x^3}{(1-x^2)^{3/2}} + \frac{x}{(1-x^2)^{1/2}} + C$



$\tan\theta = \frac{x}{\sqrt{1-x^2}}$

⑥ $$\frac{x^2-x+1}{(x^2+2x+2)(1-x)(x+2)} = \frac{Ax+B}{x^2+2x+2} + \frac{C}{1-x} + \frac{D}{x+2}$$

⑦ $$\frac{6x^2-6x-71}{(x-3)^2(x+4)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+4}$$ multiply both sides by $(x-3)^2(x+4)$

$$6x^2-6x-71 = A(x-3)(x+4) + B(x+4) + C(x-3)^2$$

Plug-in $x=3 \Rightarrow 54-18-71 = B(7) \Rightarrow B=-5$

Plug-in $x=-4 \Rightarrow 96+24-71 = C(49) \Rightarrow C=1$

Plug-in $x=0 \Rightarrow -71 = A(-12)+(-5)(4)+(1)(9) \Rightarrow A=5$

So, $$\int \frac{6x^2-6x-71}{(x-3)^2(x+4)}\,dx = \int \frac{5}{x-3} + \frac{-5}{(x-3)^2} + \frac{1}{x+4}\,dx$$

$$= 5\ln|x-3| - 5\left(-\frac{1}{x-3}\right) + \ln|x+4| + C$$

⑧ $$A_1 = \int_{\frac{\pi}{6}}^{5\pi/6} \left(\sin x - \tfrac{1}{2}\right) dx$$

$$A_2 = \int_0^{\pi/6} \sin x\,dx + \int_{\pi/6}^{5\pi/6} \tfrac{1}{2}\,dx$$

⑨ $$A = \int_{-5}^{-2} [(5)-(-x)]\,dx + \int_{-2}^{0} [(x^2+1)-(-x)]\,dx$$

⑩ 10.1 Disk / Washer

$$V = \int_1^2 \pi\left[0-(-\sqrt{y-1})\right]^2 dy + \int_2^3 \pi\left[0-(y-3)\right]^2 dy$$

Shell

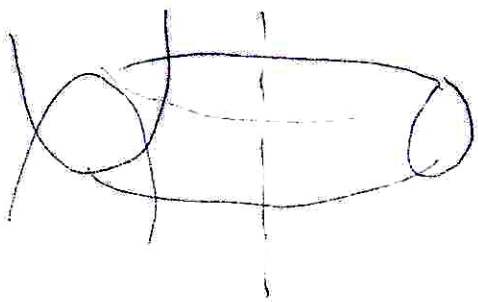
$$V = \int_{-1}^{0} 2\pi\,(0-(-x))\left((x+3)-(x^2+1)\right) dx$$

10.2 $$V = \int_{-1}^{2} \pi\left[(x+3)+1\right]^2 - \pi\left[(x^2+1)+1\right]^2 dx$$

(10.3) (i) Region C

(ii) $V = \int_{2}^{3} \pi \left[(x^2+1)^2 - (x+3)^2 \right] dx$

(12)



Shell

$V = \int_{-1}^{1} \pi (3-x)\left((-x^2+1)-(x^2-1)\right) dx$

(13) 13.1 $f(1) = 1$, $f(2) = 1.41$

13.2 $\Delta x = \frac{2.5-0.5}{2} = 1$, $x_0 = 0.5$, $x_1 = 1.5$, $x_2 = 2.5$

$y_0 = 0.25$, $y_1 = 1.31$ $x_2 = 1.44$

$T = \frac{\Delta x}{2}[y_0 + 2y_1 + y_2] = \frac{1}{2}[0.25 + 2(1.31) + 1.44]$

$=$

13.3 $\Delta x = \frac{2.5-0.5}{4} = \frac{1}{2} = 0.5$

$x_0 = 0.5$, $x_1 = 1.0$, $x_2 = 1.5$, $x_3 = 2.0$, $x_4 = 2.5$

$y_0 = 0.25$, $y_1 = 1$, $y_2 = 1.31$, $y_3 = 1.41$, $y_4 = 1.44$

$S = \frac{\Delta x}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$

$= \frac{0.5}{3}[0.25 + 4(1) + 2(1.31) + 4(1.41) + 1.44]$

$=$

(14) $y = -\sqrt{1-x^2} = -(1-x^2)^{1/2}$

$y' = -\frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}}$

$1+(y')^2 = 1 + \frac{x^2}{1-x^2} = \frac{1}{1-x^2}$

Arc length $= \int_0^1 \sqrt{1+(y')^2}\,dx = \int_0^1 \sqrt{\frac{1}{1-x^2}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)\Big|_0^1 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

(15) 15.1

$$\int_0^{+\infty} \frac{1}{x\sqrt{x}}\,dx = \int_0^1 \frac{1}{x\sqrt{x}}\,dx + \int_1^{+\infty} \frac{1}{x\sqrt{x}}\,dx$$

$$= \lim_{a\to 0^+} \int_a^1 \frac{1}{x\sqrt{x}}\,dx + \lim_{t\to\infty} \int_1^t \frac{1}{x\sqrt{x}}\,dx$$

15.2

$$\int_0^{\frac{3\pi}{2}} \frac{1}{\cos x}\,dx = \int_0^{\pi/2} \frac{1}{\cos x}\,dx + \int_{\pi/2}^{\frac{3\pi}{2}} \frac{1}{\cos x}\,dx$$

$$= \int_0^{\pi/2} \frac{1}{\cos x}\,dx + \int_{\pi/2}^{\pi} \frac{1}{\cos x}\,dx + \int_{\pi}^{\frac{3\pi}{2}} \frac{1}{\cos x}\,dx$$

$$= \lim_{b\to\frac{\pi}{2}^-} \int_0^b \frac{1}{\cos x}\,dx + \lim_{a\to\frac{\pi}{2}^+} \int_a^{\pi} \frac{1}{\cos x}\,dx + \lim_{t\to\frac{3\pi}{2}^-} \int_{\pi}^{t} \frac{1}{\cos x}\,dx$$

(16)

$$\int_0^1 x^3 \ln x\,dx = \lim_{a\to 0^+} \int_a^1 x^3 \ln x\,dx$$

$$= \lim_{a\to 0^+} \left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4\right]_a^1$$

$$= \lim_{a\to 0^+} \left[\left(0 - \frac{1}{16}\right) - \left(\frac{1}{4}a^4 \ln a - \frac{1}{16}a^4\right)\right]$$

$$= -\frac{1}{16} - 0 + 0 = -\frac{1}{16}$$

$u = \ln x \qquad dv = x^3\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{4}x^4$

$\int x^3 \ln x\,dx = \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^3\,dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

$$\lim_{a\to 0^+} a^4 \ln a = \lim_{a\to 0^+} \frac{\ln a}{\frac{1}{a^4}} = \lim_{a\to 0^+} \frac{\frac{1}{a}}{\frac{-1}{3a^3}} = \lim_{a\to 0^+} -3a^2 = 0$$