(1) 1.1 $\frac{1}{2}\left(-\frac{1}{2x-1}\right)+\frac{1}{2}\ln|2x-1| + \frac{1}{2}\frac{2^{2x-1}}{\ln 2} + c$

1.2 Let $u = 2x + x^2 \Rightarrow \frac{du}{dx} = 2+2x \Rightarrow du = 2(x+1)dx$

$\Rightarrow \frac{1}{2}du = (x+1)dx$

So, $\int \frac{x+1}{\sqrt{2x+x^2}}dx = \int \frac{1}{\sqrt{u}}\cdot\frac{1}{2}du = \frac{1}{2}\int u^{-\frac{1}{2}}du$

$= \frac{1}{2}\cdot 2\sqrt{u} + C = \sqrt{2x+x^2} + C.$

1.3 Let $u = \frac{1}{x}+2 \Rightarrow \frac{du}{dx} = -\frac{1}{x^2} \Rightarrow -du = \frac{1}{x^2}dx$

So, $\int \frac{e^{(\frac{1}{x}+2)}}{x^2}dx = \int e^{(\frac{1}{x}+2)}\cdot\frac{1}{x^2}dx = -\int e^u du$

$= -e^u + C = -e^{\frac{1}{x}+2} + C$

1.4 Let $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$

So, $\int \frac{1}{\sqrt{x}}\operatorname{cosec}(\sqrt{x})\cot(\sqrt{x})dx = \int \operatorname{cosec}(u)\cot(u)\,2\,du$

$= -2\operatorname{cosec}(u) + C = -2\operatorname{cosec}(\sqrt{x}) + C$

1.5 $\int \frac{1}{(3x+5)^2+9}dx = \frac{1}{3}\int \frac{1}{u^2+3^2}du$

let $u = 3x+5$

$\frac{du}{dx} = 3$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\cdot\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + c$

$= \frac{1}{9}\tan^{-1}\left(\frac{3x+5}{3}\right) + c$

1.6 let $u = \cos\theta \Rightarrow \frac{du}{d\theta} = -\sin\theta \Rightarrow -du = \sin\theta\, d\theta$

so, $\int \cos(\cos\theta)\sin\theta\, d\theta = -\int \cos(u)\, du = -\sin u + c$

$= -\sin(\cos\theta) + c$

(2) $\int x\cos(9x)\,dx = \frac{1}{9}x\sin(9x) - \frac{1}{9}\int \sin(9x)\,dx$

let $u = x$ $\quad dv = \cos(9x)dx$

$du = dx$ $\quad v = \frac{1}{9}\sin(9x)$

$= \frac{1}{9}x\sin(9x) + \frac{1}{81}\cos(9x) + c$

(3) $\int x\, 7^x \ln 7\, dx = x7^x - \int 7^x\, dx$

let $u = x$ $\quad dv = 7^x \ln 7\, dx$

$du = dx$ $\quad v = 7^x$

$= x7^x - \frac{7^x}{\ln x} + c$

(4) $\int \sin^2(5x)\cos^2(5x)\,dx = \int \left[\frac{1}{2}\sin(10x)\right]^2 dx$

$= \int \frac{1}{4}\sin^2(10x)\,dx = \frac{1}{8}\int (1 - \cos(20x))\,dx$

$= \frac{1}{8}x - \frac{1}{160}\sin(20x) + c$

(5) $\int 2\sin(9\theta)\cos(2\theta)\,d\theta = \int \sin(11\theta) + \sin(7\theta)\,d\theta$

$= \frac{-1}{11}\cos(11\theta) - \frac{1}{7}\cos(7\theta) + C$

(6) $\sqrt{4x^2+9} = \sqrt{9\left(\frac{4}{9}x^2+1\right)} = 3\sqrt{\left(\frac{2}{3}x\right)^2+1}$

Let $\frac{2}{3}x = \tan\theta \Rightarrow \sqrt{\left(\frac{2}{3}x\right)^2+1} = \sqrt{\tan^2\theta+1} = \sec\theta$

$x^4 = \left(\frac{3}{2}\tan\theta\right)^4 = \frac{81}{16}\tan^4\theta$

$\frac{2}{3}\,dx = \sec^2\theta\,d\theta$

$$\Rightarrow \int \frac{\sqrt{4x^2+9}}{x^4}\,dx = \int \frac{\sec\theta}{\frac{81}{16}\tan^4\theta}\sec^2\theta\,d\theta$$

$$= \frac{16}{81}\int \frac{\sec^3\theta}{\tan^4\theta}\,d\theta = \frac{16}{81}\int \frac{1}{\cos^3\theta}\cdot\frac{\cos^4\theta}{\sin^4\theta}\,d\theta$$

$$= \frac{16}{81}\int \frac{\cos\theta}{\sin^4\theta}\,d\theta$$

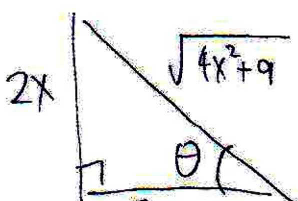
let $u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{16}{81}\int \frac{1}{u^4}\,du = \frac{16}{81}\cdot\left(-\frac{1}{3}u^{-3}\right) + C$$

$$= -\frac{16}{243}u^{-3} + C = -\frac{16}{243}\frac{1}{\sin^3\theta} + C$$

$\frac{2x}{3} = \tan\theta$



$\sin\theta = \frac{2x}{\sqrt{4x^2+9}}$

$$= -\frac{16}{243}\frac{1}{\left(\frac{2x}{\sqrt{4x^2+9}}\right)^3} + C$$

$$= -\frac{16}{243}\frac{(4x^2+9)^{3/2}}{(2x)^3} + C$$

$$= -\frac{2}{243}\frac{(4x^2+9)^{3/2}}{x^3} + C$$

(7) 7.1 $$\frac{2x^3-2x^2+5x+1}{(x-1)^2(x^2+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2}$$

7.2 $$\frac{2x^3-1}{x(x^2-2x+2)^2} = \frac{A}{x} + \frac{Bx+C}{x^2-2x+2} + \frac{Dx+E}{(x^2-2x+2)^2}$$

(8) $$\frac{3x^2+2x+2}{(x-1)(x^2+2x+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+4}$$ Multiply both sides by $(x-1)(x^2+2x+4)$

$$\Rightarrow \quad 3x^2+2x+2 = A(x^2+2x+4) + (Bx+C)(x-1)$$

$$3x^2+2x+2 = Ax^2+2Ax+4A + Bx^2+Cx-Bx-C$$

$$3x^2+2x+2 = (A+B)x^2 + (2A+C-B)x + (4A-C)$$

$$\Rightarrow \begin{cases} A+B=3 \\ 2A+C-B=2 \\ 4A-C=2 \end{cases} \Rightarrow \begin{matrix} B=3-A \\ \\ C=4A-2 \end{matrix} \Rightarrow 2A+(4A-2)-(3-A)=2$$

$$\Rightarrow A=1, \quad B=2, \quad C=2$$

$$\int \frac{3x^2+2x+2}{(x-1)(x^2+2x+4)}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{2x+2}{x^2+2x+4}\,dx$$

$$= \ln|x-2| + \ln|x^2+2x+4| + C$$

(9) $$\text{Area} = \int_2^5 -\frac{e}{y}\,dy = \Big[-e\ln|y|\Big]_2^5$$

$$= -e(\ln 5 - \ln 2) = -e\ln\frac{5}{2}$$

(10) $$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\pi/2} (\sin x - \cos x)\,dx$$

$$+ \int_{\frac{\pi}{2}}^{3\pi/4} \left(\cos x - \left(x - \tfrac{\pi}{2}\right)\right) dx$$

(11) 11.1 Shell $$V = \int_{-1}^{0} 2\pi(1-x)\sqrt{x+1}\,dx + \int_0^1 2\pi(1-x)e^{-x}\,dx$$

Disk $$V = \int_0^{1/e} \pi\left(1-(y^2-1)\right)^2 dy + \int_{1/e}^{1} \left[\pi(-\ln x)^2 - \pi(y^2-1)^2\right] dx$$

11.2 Shell $$V = \int_{\frac{1}{e}}^{1} 2\pi(y)\left(1-(-\ln x)\right) dy + \int_1^{\sqrt{2}} 2\pi(y)\left(1-(y^2-1)\right) dy$$

Disk $$V = \int_0^1 \pi\left[\sqrt{x+1}\right]^2 - \pi\left[e^{-x}\right]^2\,dy$$

(12) $$y = \frac{1}{3}\left(x^{3/2} - 3x^{\frac{1}{2}} + 2\right)$$

$$y' = \frac{1}{3}\left(\frac{3}{2}x^{1/2} - \frac{3}{2}x^{-\frac{1}{2}}\right) = \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$$

$$1+(y')^2 = 1 + \frac{1}{4}\left(x - 2 + x^{-1}\right) = \frac{1}{4}x + \frac{1}{2} + \frac{1}{4x} = \left(\frac{1}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}\right)^2$$

$$\text{Arc length} = \int_0^1 \sqrt{1+(y')^2}\,dx = \int_0^1 \sqrt{\left(\frac{1}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}\right)^2}\,dx$$

$$= \int_0^1 \frac{1}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}\,dx = \int_0^1 \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\left[\frac{2}{3}x^{3/2} + 2x^{\frac{1}{2}}\right]_0^1 = \frac{1}{2}\left[\frac{2}{3} + 2\right] = \frac{4}{3}$$

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(13.1) $T = \frac{\Delta x}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right] = \frac{0.4}{2}\left[0.18 + 2(0.47) + 2(0.69) + 2(0.88) + 1.03\right]$

$=$

$\Delta x = \frac{2.8 - 1.2}{4} = \frac{1.6}{4} = 0.4$

$x_0 = 1.2$	$y_0 = f(1.2) = 0.18$
$x_1 = 1.2 + 0.4 = 1.6$	$y_1 = f(1.6) = 0.47$
$x_2 = 1.6 + 0.4 = 2.0$	$y_2 = f(2.0) = 0.69$
$x_3 = 2.0 + 0.4 = 2.4$	$y_3 = f(2.4) = 0.88$
$x_4 = 2.8$	$y_4 = f(2.8) = 1.03$

(13.2)

$\Delta x = \frac{2.8 - 1}{6} = \frac{1.8}{6} = 0.3$

$x_0 = 1$	$y_0 = f(1) = 0$
$x_1 = 1 + 0.3 = 1.3$	$y_1 = f(1.3) = 0.26$
$x_2 = 1.3 + 0.3 = 1.6$	$y_2 = f(1.6) = 0.47$
$x_3 = 1.6 + 0.3 + 1.9$	$y_3 = f(1.9) = 0.64$
$x_4 = 1.9 + 0.3 = 2.2$	$y_4 = f(2.2) = 0.79$
$x_5 = 2.2 + 0.3 = 2.5$	$y_5 = f(2.5) = 0.92$
$x_6 = 2.8$	$y_6 = f(2.8) = 1.03$

$$S = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right]$$

$$= \frac{0.3}{3}\left[0 + 4(0.26) + 2(0.47) + 4(0.64) + 2(0.79) + 4(0.92) + 1.03\right]$$

$=$

(14) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t \to 1^-} \int_0^t \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t \to 1^-} \sin^{-1}(x)\Big|_0^t = \lim_{t \to 1^-} \sin^{-1}(t) - \sin^{-1}(0) = \frac{\pi}{2}$

(15) 15.1 $\cancel{\int_{-1}^{+\infty} \frac{1}{(x+2)\sqrt{x}}\,dx =}$

$$\int_1^{+\infty} \frac{1}{(x+2)\sqrt{x}}\,dx = \lim_{t \to \infty} \int_1^t \frac{1}{(x+2)\sqrt{x}}\,dx$$

15.2 $\int_0^{\pi/2} \frac{1}{x\cos x}\,dx = \int_0^{\pi/4} \frac{1}{x\cos x}\,dx + \int_{\pi/4}^{\pi/2} \frac{1}{x\cos x}\,dx$

$$= \lim_{a \to 0^+} \int_a^{\pi/4} \frac{1}{x\cos x}\,dx + \lim_{b \to \frac{\pi}{2}^-} \int_0^b \frac{1}{x\cos x}\,dx$$