

## แบบฝึกหัด

1. 
$$\int \frac{2(x^4 + 1)}{(x - 1)(x + 1)^2(x^2 + 1)^2} dx$$

2. กำหนดให้  $n = 4$  จงใช้หลักเกณฑ์เชิงสี่เหลี่ยมคางหมูเพื่อประมาณ

ค่าของ  $\int_0^2 \sqrt{1 + \frac{\sin(\pi x)}{2}} dx$  โดยที่  $\sqrt{0.5} \approx 0.707$  และ  $\sqrt{1.5} \approx$

1.225

$$\textcircled{1} \quad \frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

$$2(x^4+1) = A(x+1)^2(x^2+1)^2 + B(x-1)(x+1)(x^2+1)^2 + (Dx+E)(x-1)(x+1)^2(x^2+1) + (Fx+G)(x-1)(x+1)^2$$

$$\Rightarrow A = \frac{1}{4}, \quad B = -\frac{1}{4}, \quad C = -\frac{1}{2}, \quad D = E = 0, \quad F = 1, \quad G = -1$$

$$\Rightarrow \frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{2}}{(x+1)^2} + \frac{x-1}{(x^2+1)^2}$$

$$\int \frac{2(x^4+1)}{(x-1)(x+1)^2(x^2+1)^2} dx = \int \frac{\frac{1}{4}}{x-1} dx + \int \frac{-\frac{1}{4}}{x+1} dx + \int \frac{-\frac{1}{2}}{(x+1)^2} dx + \int \frac{x-1}{(x^2+1)^2} dx$$

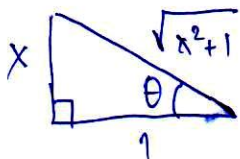
$$= \frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| + \frac{1}{2} \cdot \frac{1}{x+1} + \int \frac{x-1}{(x^2+1)^2} dx$$

$$\int \frac{x-1}{(x^2+1)^2} dx = \int \frac{x}{(x^2+1)^2} dx - \int \frac{1}{(x^2+1)^2} dx$$

$$= \frac{1}{2} \left( -\frac{1}{x^2+1} \right) - \int \frac{1}{\sec^4 \theta} \sec^2 \theta d\theta$$

$$x = \tan \theta \Rightarrow x^2+1 = \sec^2 \theta$$

$$dx = \sec^2 \theta d\theta$$



$$\sin \theta = \frac{x}{\sqrt{x^2+1}}$$

$$\cos \theta = \frac{1}{\sqrt{x^2+1}}$$

$$= \int \cos^2 \theta d\theta$$

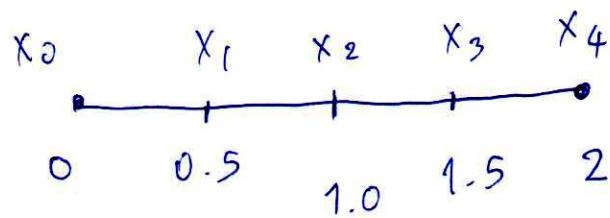
$$= \int \frac{1}{2} d\theta + \int \frac{1}{2} \cos 2\theta d\theta$$

$$= \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta + C$$

$$= \frac{1}{2} \theta + \frac{1}{2} \sin \theta \cos \theta + C$$

$$= \frac{1}{2} \tan^{-1} x + \frac{1}{2} \frac{x}{\sqrt{x^2+1}} + C$$

(2)



$$\Delta x = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

$$y_0 = \sqrt{1 + \frac{\sin(0)}{2}} = \sqrt{1} = 1$$

$$y_1 = \sqrt{1 + \frac{\sin \frac{\pi}{2}}{2}} = \sqrt{1.5} \approx 1.225$$

$$y_2 = \sqrt{1 + \frac{\sin \pi}{2}} = \sqrt{1} = 1$$

$$y_3 = \sqrt{1 + \frac{\sin \frac{3\pi}{2}}{2}} = \sqrt{0.5} \approx 0.707$$

$$y_4 = \sqrt{1 + \frac{\sin 2\pi}{2}} = \sqrt{1} = 1$$

$$T = \frac{\Delta x}{2} [y_0 + 2y_1 + 2y_2 + 2y_3 + y_4]$$

$$= \frac{0.5}{2} [1 + 2(1.225) + 2(1) + 2(0.707) + 1]$$

$$= 1.966$$