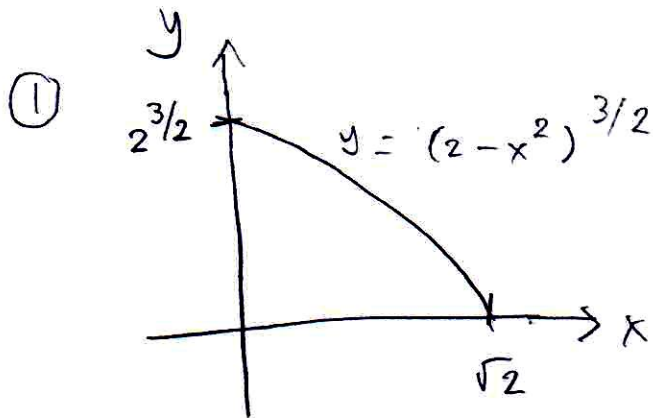


แบบฝึกหัด

1. จงหาพื้นที่ใต้กราฟของ $y = (2 - x^2)^{3/2}$ ในจตุภาคที่หนึ่ง

2.
$$\int \frac{2x + 1}{x^2 - 3x + 2} dx$$



$$\text{Area} = \int_0^{\sqrt{2}} (2 - x^2)^{3/2} dx$$

Consider $\int (2 - x^2)^{3/2} dx$

Let $x = \sqrt{2} \sin \theta \Rightarrow dx = \sqrt{2} \cos \theta d\theta$

$$\begin{aligned} (2 - x^2)^{3/2} &= (2 - 2\sin^2 \theta)^{3/2} \\ &= 2^{3/2} (1 - \sin^2 \theta)^{3/2} \\ &= 2^{3/2} \cos^3 \theta \end{aligned}$$

When $x = 0 \Rightarrow 0 = \sqrt{2} \sin \theta \Rightarrow \theta = 0$

When $x = \sqrt{2} \Rightarrow \sqrt{2} = \sqrt{2} \sin \theta \Rightarrow \sin \theta = 1$
 $\Rightarrow \theta = \frac{\pi}{2}$

So,

$$\begin{aligned} \int (2 - x^2)^{3/2} dx &= \int 2^{3/2} \cos^3 \theta \cdot \sqrt{2} \cos \theta d\theta \\ &= 4 \int \cos^4 \theta d\theta \\ &= 4 \int \left(\frac{1 + \cos 2\theta}{2} \right)^2 d\theta \\ &= \int (1 + 2\cos 2\theta + \cos^2 2\theta) d\theta \\ &= \theta + \sin 2\theta + \int \cos^2 2\theta d\theta \\ &= \theta + \sin 2\theta + \frac{1}{2} \int (1 + \cos 4\theta) d\theta \\ &= \theta + \sin 2\theta + \frac{1}{2} \theta + \frac{1}{8} \sin 4\theta + C \\ &= \frac{3}{2} \theta + \sin 2\theta + \frac{1}{8} \sin 4\theta + C \end{aligned}$$

Therefore, area = $\int_0^{\sqrt{2}} (2 - x^2)^{3/2} dx$

$$= \left[\frac{3}{2} \theta + \sin 2\theta + \frac{1}{8} \sin 4\theta \right]_0^{\frac{\pi}{2}}$$

$$= \frac{3}{2} \left[\frac{\pi}{2} - 0 \right] + [\sin \pi - \sin 0] + \frac{1}{8} [\sin 2\pi - \sin 0]$$

$$= \frac{3\pi}{4}$$

② $\int \frac{2x+1}{x^2-3x+2} dx = \int \frac{2x+1}{(x-2)(x-1)} dx$

$$\frac{2x+1}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$$

$$\frac{2x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$2x+1 = A(x-2) + B(x-1)$$

$$\text{Plug-in } x=1 \Rightarrow 2+1 = A(1-2) \Rightarrow A = -3$$

$$\text{Plug-in } x=2 \Rightarrow 4+1 = B(2-1) \Rightarrow B = 5$$

$$\text{So, } \frac{2x+1}{(x-1)(x-2)} = \frac{-3}{x-1} + \frac{5}{x-2}$$

$$\begin{aligned} \int \frac{2x+1}{x^2-3x+2} dx &= \int \frac{-3}{x-1} dx + \int \frac{5}{x-2} dx \\ &= -3 \ln|x-1| + 5 \ln|x-2| + C \end{aligned}$$