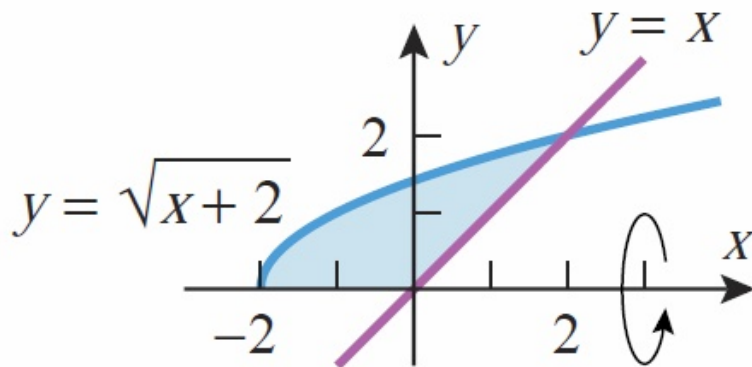
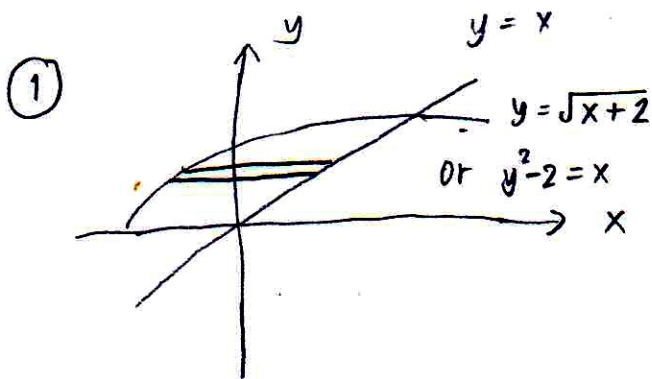


แบบฝึกหัด

1. จงใช้วิธีแบบเปลือกทรงกระบอกเพื่อหาปริมาตรของทรงตัน



2. จงหาความยาวเส้นโค้ง $y = \frac{2\sqrt{2}}{3}x^{3/2} - 1$, $0 \leq x \leq 1$



$$\text{Volume} = \int_0^2 2\pi y (y - (y^2 - 2)) dy$$

$$= 2\pi \int_0^2 y^2 - y^3 + 2y dy$$

$$= 2\pi \left[\frac{1}{3}y^3 - \frac{1}{4}y^4 + y^2 \right]_0^2 = 2\pi \left[\frac{8}{3} - 4 + 4 \right] = \frac{16\pi}{3}$$

②

$$y = \frac{2\sqrt{2}}{3} x^{3/2} - 1$$

$$\frac{dy}{dx} = \sqrt{2} x^{1/2}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 2x$$

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + 2x} dx$$

$$= \int_1^3 \sqrt{u} \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int_1^3 u^{1/2} du$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} \Big|_1^3$$

$$= \frac{1}{3} u^{3/2} \Big|_1^3$$

$$= \frac{1}{3} \left[3^{3/2} - 1^{3/2} \right]$$

$$= \frac{1}{3} [3\sqrt{3} - 1]$$

$$= \sqrt{3} - \frac{1}{3}$$

let $u = 1 + 2x$

$$\frac{du}{dx} = 2$$

$$\frac{1}{2} du = dx$$

When $x = 0, u = 1$

$x = 1, u = 1 + 2 = 3$