

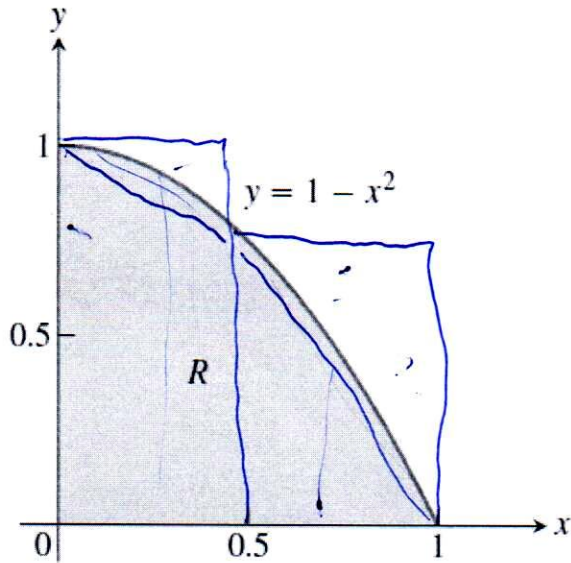
แบบฝึกหัด

1. $\int \sin^5 \left(\frac{\theta}{2} \right) \cos \left(\frac{\theta}{2} \right) d\theta.$

2. $\int x(x-1)^{10} dx.$

3. $\int \sqrt{\frac{2x-1}{x^5}} dx.$

4. จงประมาณพื้นที่ของอาณาบริเวณ R จากตัวอย่าง 4 ข้างต้น



① Let $u = \sin \frac{\theta}{2}$

so, $\frac{du}{d\theta} = \cos \frac{\theta}{2} \cdot \frac{1}{2}$

$\frac{2 du}{\cos \frac{\theta}{2}} = d\theta$

$\int \sin^5 \left(\frac{\theta}{2} \right) \cos \left(\frac{\theta}{2} \right) d\theta$

$= \int u^5 \cdot \cos \left(\frac{\theta}{2} \right) \cdot \frac{2 du}{\cos \left(\frac{\theta}{2} \right)}$

$= 2 \int u^5 du$

$= 2 \cdot \frac{u^6}{6} + C = \frac{1}{3} \sin^6 \frac{\theta}{2} + C$

② Let $u = x-1$
($u+1 = x$)

so, $\frac{du}{dx} = 1$

$du = dx$

$\int x(x-1)^{10} dx = \int (u+1)u^{10} du$

$= \int u^{11} + u^{10} du$

$= \frac{1}{12} u^{12} + \frac{1}{11} u^{11} + C$

$= \frac{1}{12} (x-1)^{12} + \frac{1}{11} (x-1)^{11} + C$

③ $\int \sqrt{\frac{2x-1}{x^5}} dx = \int \sqrt{\frac{2x-1}{x^4 \cdot x}} dx = \int \frac{1}{x^2} \sqrt{2-\frac{1}{x}} dx$

Let $u = 2 - \frac{1}{x}$

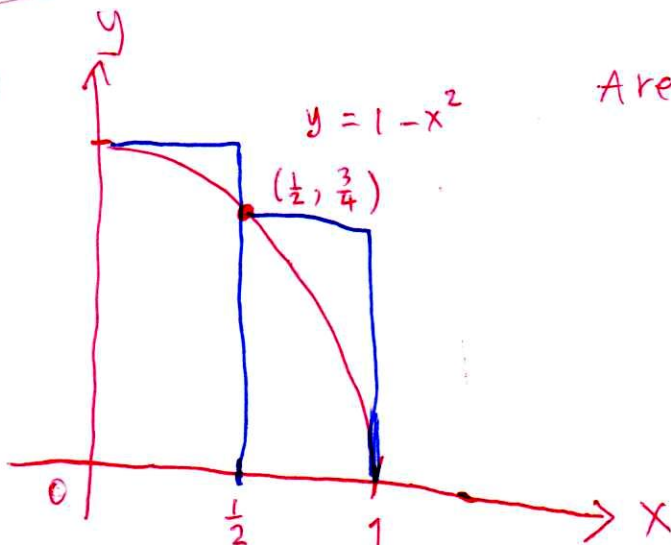
so $\frac{du}{dx} = \frac{1}{x^2}$

$x^2 du = dx$

$= \int \frac{1}{x^2} \sqrt{u} \cdot x^2 du = \int \sqrt{u} du$

$= \frac{2}{3} u^{3/2} + C = \frac{2}{3} \left(2 - \frac{1}{x} \right)^{3/2} + C$

④



Area \approx $\square + \square$

$= \left(\frac{1}{2} \right) (1) + \left(\frac{1}{2} \right) \left(\frac{3}{4} \right)$

$= \frac{1}{2} + \frac{3}{8}$

$= \frac{7}{8}$