

In-class exercises

① Find y' if $y = \frac{4}{\cos x} + \frac{x}{\tan x}$

Solution

$$y = \frac{4}{\cos x} + \frac{x}{\tan x} = 4\sec x + x\cot x$$

$$\begin{aligned} y' &= 4\sec x \tan x + x(\cot x)' + \cot x(x)' \\ &= 4\sec x \tan x + x(-\csc^2 x) + \cot x \end{aligned}$$

② Find tangent line of $y = x - \cot x$ at $x = \frac{\pi}{4}$.

Solution

$$f(x) = x - \cot x$$

$$f'(x) = 1 + \csc^2 x$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - \cot \frac{\pi}{4} = \frac{\pi}{4} - 1$$

$$f'\left(\frac{\pi}{4}\right) = 1 + \csc^2 \frac{\pi}{4} = 1 + 2 = 3$$

$$\text{So, } (x_0, y_0) = \left(\frac{\pi}{4}, \frac{\pi}{4} - 1\right)$$

$$m = 3$$

Equation of tangent line: $y - y_0 = m(x - x_0)$

$$\boxed{y - \left(\frac{\pi}{4} - 1\right) = 3\left(x - \frac{\pi}{4}\right)}$$