Conversation with a leading mathematician of the century

Professor Isadore M. Singer

Massachusetts Institute of Technology (MIT), USA

Professor Isadore M. Singer shared the 2004 Abel Prize with Sir Michael Francis Atiyah in May 2004. The prize has been linked to the Nobel Prize, but for Mathematics. After he received the prize for a few months, he was very kind by spending his valuable time to let Atichart Kettapun and Charlie Lertpattarapong interview him at his office at MIT. This interview was transcribed by Pramoo Khungurn.

Kettapun: What is the inspiration that made you discovered the famous "Index Theory"?

Singer: There was an important question in *topology*[1]: why, for certain *manifolds*[2] (they're called Spin manifolds), an integral, called $\hat{A}$-genus, was an integer? This question is motivated by the work of Hirzebruch on the signature of a manifold, where the integral of the L-genus was an integer, and the Atiyah-Hirzebruch Riemann Roch formula. We worked on that question and came up with a surprising answer: it was an integer because it was the index of the *Dirac operator* [3] for Spin manifolds. We quickly derived the formula for the index of the Dirac operators coupled to *vector bundles*[4], but it took us some time to prove the theorem. To get the answer in terms of the Dirac operator required a new combination of topology, geometry, and analysis beyond Hodge Theory. That broke some barriers between topology, geometry, and analysis that I would say, made for a revolution in modern mathematics

Kettapun: That's very interesting because it brings many fields together.

Singer: Yes, I think that was one of the most fascinating and exciting parts of what Sir Michael and I did.

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[1] The study of the properties of geometric figures or solids that are not changed by homeomorphisms, such as stretching or bending. Donuts and picture frames are topologically equivalent, for example. (Link)

[2] A topological space or surface. (Link)

[3] A particular mapping in Mathematics. (Link)

[4] A particular mapping in Mathematics. (Link)

Kettapun: Only few mathematicians are successful like you and then I think many people would like to know the important factors that made you highly succeed in your work

Singer: Well, who's to say... **I think Sir Michael and I are both very broad-based mathematicians. We come from different backgrounds, but we share the sense of the unity of mathematics; we don't see barriers between fields.** We are each free-willing, trying out different possibilities and being more flexible than most mathematicians. I think that helped us a great deal on this particular problem.

Kettapun: I think many people focus on the particular fields they are working on rather than seeing alternatives.

Singer: Neither of us was narrowly-based in mathematics; even though we came from different backgrounds we could communicate easily.

Lertpattarapong: Do you draw ideas from other fields, like physics, biology, and chemistry, or just pure mathematics?

Singer: In our case, physics was influential because we used the Dirac operator. I remind you of Dirac's famous equation for the spinning electron. Some 20 years later there were many applications of the index theorem to physics.

Kettapun: Actually, many smart students tend to choose popular subjects, like medicine, law, engineering and other fields that they can make more money. Would you like to say something to convince more smart students to be more interested in mathematics or even pure science?

Singer: **I think "able" students should be allowed to experiment in different fields and see what they enjoy doing. If life is kind to them, following what they find exciting will lead to success.** Students should look at many different possibilities; hopefully their school permits lots of choice. Practicing medicine or law are traditional ways of insuring economic security, and are satisfying ways of contributing to society. But it appears that many doctors and lawyers are not happy because they do not enjoy their work. **Those of us in mathematics who enjoy doing mathematics are lucky; all day long, we're doing something we're excited about.**

As an undergraduate, I couldn't decide whether I wanted to major in English or physics. I soon realized that my fellow students in English understood poetry instantly, while I had to work hard to understand a good poem. I understood physics instantly, while others struggled with the subject. It was clear my talent was in science, not in literature.

At that time, I didn't know one could have a career as a research mathematician, although Eilenberg and Steenrod were at the University of Michigan. When I graduated and joined the US Army, I took math textbooks with me to cement my understanding of quantum mechanics and relativity. When I returned to graduate school after World War II, I majored in mathematics, planning to return to physics after a year.

But I never left mathematics. Luckily, when I received my Ph.D. (1950), there were many openings at universities, though not at the major ones. It was possible to live modestly and enjoy teaching and doing research. I was fortunate in having many choices. I work hard these days to help young people find the opportunities they need to develop their talents.

I don't know the educational system in Thailand, but **I would hope that there is room for talented people to explore and finally choose something they really love to do. I believe the country on the whole would benefit by that freedom of choice.**

Kettapun: Actually, there is a project to support Thai students to study Mathematics and pure science, which is called the **"Promotion of Science & Technology for Talents Project (DPST)"**. It supports scholarships and training for some talented students to study in those fields from grade 10 up to doctoral degree. That is one thing that the Thai government is doing and it is a really long program.

Singer: I'm glad to hear that. MIT reflects the sense that mathematics and the core sciences, physics, and chemistry, and, today, biology, are the foundations for serious later endeavors. Our freshmen and sophomore courses are based on that philosophy, and I think it's the correct philosophy. Our students work hard in these core courses knowing they will be useful whatever they choose to do.

Kettapun: What do you think of mathematics in the next few decades related to other sciences or other fields including social sciences?

Singer: I think the future is different in each one of the subjects you mentioned. Let me make a general comment. I've known people in many different fields, and it's quite noticeable that someone with mathematical insights, whether by training or talent, shows a special analytical viewpoint, that is quite useful. There is a penetration of the issues lacking in those not trained in mathematics or not having talent for thinking mathematically.

Kettapun: I remember the time I studied an undergraduate economics course. When a professor explained things in mathematics, I felt that it was not quite realistic. Do you feel in that way? I think in social sciences, like human behaviors and others, they are more complicated. So, I think it is harder to use mathematics in those fields.

Singer: I agree with you that human beings are much more complicated than the simple effective models in the applications of mathematics to the hard sciences. We can isolate a moving particle or a planet, and can describe its motion; human beings are much more complex. **Scientists who use mathematics in the social or behavioral sciences must be careful with the conclusions drawn from simplistic mathematical models.** Still, statistical models, for example, can be very effective. Using our knowledge of the human genome to find and study defective genes and combinations of genes is based on sophisticated statistical methods. These refined models work very well.

In economics, I find some of the uses of mathematics naive. But there are many smart people in the subject who know mathematics and use it judiciously in their analysis of economic problems. Mathematics can be misused.

Kettapun: Do you think a lot of people misuse mathematics?

Singer: I don't know about a lot, but certainly some. They go overboard, getting formulas applicable in simple situations, and applying them to more general ones. That doesn't work well.

Kettapun: I would like to thank you very much for spending your time for this interview.

Singer: I'd like to encourage young people in Thailand who show some mathematical talent to pursue mathematics; it is useful in so many places. I am confident their talent and training will eventually have important applications. MIT is an example of a wonderful combination of science, technology, and engineering. As you may have noticed while you are here, we say, **"The place is buzzing!." I can't walk from the Red Line at Kendall Square (MIT subway station) to my office without encountering something new. That's very exciting. That's what a university should be like. I hope that young students in Thailand, in their own way, can experience that kind of excitement in science and engineering, and that they will become part of it.**

Kettapun: That's really a very encouraging speech. Thank you.

Singer: Good luck!

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First Publishing at www.siamsewana.org on May 15, 2005