1. Let $F : R^n \to R^n$ be a smooth function. It is desired to solve the equation $F(x) = 0$ using Newton's method.
a) State Newton's method.
b) Suppose $n = 2$, and $F(x) = \begin{bmatrix} x_1^2 + x_2 \\ x_1 + x_2^2 \end{bmatrix}$.
If the initial guess is $x^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, what is the first iterate of Newton's method?
c) Give some conditions on the function F and the initial guess x^0 such that the iterates x^k are all well defined and the sequence of iterates converges quadratically to a zero x^* of F.

2. Consider the initial value problem $y' = f(x, y), y(0) = y_0$. To solve it, we let $x_i = ih$ be a set of uniformly spaced mesh points, and we write

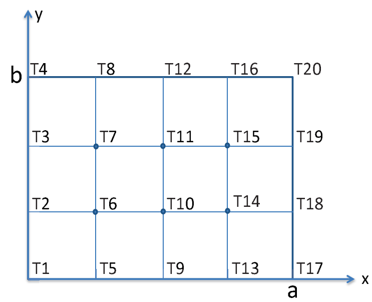
$$y_{p+1} = y_{p-1} + \int_{x_{p-1}}^{x_{p+1}} f(x, y(x))\, dx.$$

We approximate the integrand with Lagrange interpolating polynomial interpolating $f(x, y(x))$ at the points $x = x_p, x_{p-1}, x_{p-2}$.
a) Determine the coefficients of the multistep formula that results from this approximation.
b) Determine whether the multistep formula is stable.
c) Determine the order of accuracy of this multistep formula.

3. Determine the linear system for solving the following Laplace's equation of $T(x, y)$,

$$\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y} = 0, \quad 0 \le x \le a, 0 \le y \le b$$
$$T(0, y) = g_1(y), \quad \frac{\partial T(a, y)}{\partial x} = g(y),$$
$$T(x, 0) = f_1(x), \quad T(x, b) = f_2(x),$$

using central difference approximation on both x and y directions. The discretization of the domain is given in the picture using the uniform spacing h for discretization in both x and y directions.



4. Given boundary value problem $y'(x) = f(x, y), \quad y(0) = A, y(b) = B$.

   (a) Suggest **two numerical methods** for solving the given problem.

   (b) Write **pro and cons** of the numerical technique given in (a).