Numerical Optimization

A Workshop
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Session:

Introduction and Modeling

Successful practice of optimization depends on good modeling skills, understanding of efficient and reliable algorithms, selection and use of appropriate software, and careful interpretation of results.

Emphasis

Modeling: Framing a good optimization model from a descriptive problem.

Algorithms: Intuitive understanding, common sense and geometrical concepts of how algorithms work.

Computation: Emphasis on computer solutions, Software to be used: EXCEL SOLVER and LINGO, MATLAB, Software for Dynamic Optimization

Interpretation: Based on conceptual/geometric understanding of methods; post-optimality and sensitivity analysis

Applications: Engineering designs, Statistical Decision Making, Systems Biology, Medical Imaging and Treatment Planning, Smart Energy Systems and Smart Power Grid

Goal

Sufficient understanding and computational skills in Numerical Optimization for

- Unconstrained and constrained continuous-variable problems LP, NLP
- Discrete Optimization--IP
- Dynamic Optimization
- Large-scale and Multiobjective Optimization

Course Outline

Session **Topic**

1.1 ***Modeling and Computer Solution***

– Framing engineering problems as optimization problems, and developing appropriate optimization models

– Common-sense optimization

– Intro to EXCEL, LINGO, MATLAB modelers and optimizers,

Course Outline

Session **Topic**

1.2 ***Numerical Optimization for Continuous Problems: Unconstrained methods***

– Unconstrained methods for smooth problems
  - Direction-finding (Steepest-Descent, Newton's and variations, Quasi-Newton, Conjugate-Direction)
  - Line search methods
  - Applications: linear and nonlinear least-square problems. Neural Networks

– Unconstrained methods for nonsmooth problems
  - Nelder-Mead's Simplex method
  - Genetic Algorithm
  - Simulated Annealing

– Use of MATLAB to implement the algorithms

– Application on Engineering Design Cases

Course Outline

Session **Topic**

2.1 ***Numerical Optimization for Continuous Problems: Constrained Convex Optimization***

- Overview of methods for LP and Convex Problems
  - **Simplex Method and Interior Point methods**
- Other NLP optimization methods
  - **Successive Quadratic Programming (SQP)**
    - Quadratic programming
    - Ideas and Algorithms
    - Use of SQP in MATLAB, LINGO or PSPv.5 to solve smooth constrained nonlinear programs
  - Generalized reduced gradients (GRG2)
    - Ideas and algorithms: Use of GRG2 in EXCEL's SOLVER
- Implementation and Application Issues

Course Outline

Week **Topic**

2.2 ***Optimization of dynamic systems and nonsmooth cases***

- Indirect Methods: Euler-Lagrange Eqn, Pontryagin's Minimum Principle
- Dynamic Programming,
- Direct methods through optimality conditions leading to nonsmooth optimization Nonlinear programming methods: Sparse SQP and SNOPT and Dynamic Programming (DP)

Course Outline

Session **Topic**

3 ***Discrete/Combinatorial Optimization***

- Applications and formulating LIP , Combinatorial Optimization models
- Overview of Methods for Combinatorial/Integer programs
  - Exact methods
  - Constraint Programming
  - Heuristics
  - Approximation methods such as Simulated Annealing and Genetic Algorithms

Course Outline

Week **Topic**

4.1 ***Large-Scale, Global Optimization***

- Partitioning-Decomposition techniques
- Lagrangian Relaxation, Surrogate Relaxation
- Decomposition-Coordination methods
- Approximation strategies

Course Outline

Session **Topic**

4.2 ***Multiobjective Optimization***

- Overview of key concepts (Pareto-optimality, utility functions, etc.)
- Overview of methods for generating Pareto-optimal solutions (weighting approach , constraint approach)
- Practical methods (Goal programming, Analytic Hierarchy Process –AHP)
- Application on Engineering Design Cases
- Wrap-up/Review

Model Building

Do the right thing

Do the thing right

Model Building

Purpose:

- To show how to formulate algebraic optimization models
- To show how to translate into a spreadsheet (EXCEL) model and to use SOLVER to find an optimal solution and to do sensitivity analysis
- To show how to use MATLAB-based modeler and solvers

Model Building

Optimization Problem:

➔ Choose a best alternative from among those that are available.

Model Building

Alternative---Decision Variables

- **We must be able to describe an alternative mathematically in an optimization model.**
- **The first thing we do in building an optimization model is TO DEFINE AN APPROPRIATE SET OF DECISION VARIABLES**

Model Building

Examples:

- **How to invest $5,000 on four different types of stocks**
- **An alternative is a specific combination of the amount invested in each stock**
- **e.g. an alternative x_i = fraction of $5000 invested in Stock *i*: For example, for $x_1 = 0.4$; $x_2 = 0.5$; $x_3 = 0.1$; $x_4 = 0$; imply**

  $2,000 on stock 1, $2,500 on stock 2, and $500 on stock 3, and $0 on stock 4

Model Building

Best:---Objective Function

- **We must next know how to compare those alternatives, so that the "best" alternative can be chosen.**
- **Identify performance criteria or "objective functions"**
- **The problem becomes one of choosing the values of the decision variables that "optimize" the objective function.**

Model Building

Examples:

- **How to invest $5,000 on four different types of stocks**
- **An objective: Maximize the expected return on the investment:**

$$\max\ f(x) = 5000(r_1x_1 + r_2x_2 + r_3x_3 + r_4x_4)$$

or simply

$$\max\ f(x) = r_1x_1 + r_2x_2 + r_3x_3 + r_4x_4$$

Model Building

2nd step in building an optimization model :

- **Identify the objective function to be optimized**
- **Express it as a function of decision variables**

Model Building

Available ---Constraints

- **Set of alternatives is usually not unrestricted**
- **Certain limitations or requirements will restrict our choice to within the constraint set or "the set of feasible solutions".**

Model Building

In Stock Selection Example:

- **Budget cannot exceed \$5,000**

$$x_1 + x_2 + x_3 + x_4 \le 1$$

- **Risk level cannot exceed \$0.5/1\$ invested**

$$\sigma_1^2 x_1^2 + \sigma_2^2 x_2^2 + \sigma_3^2 x_3^2 + \sigma_4^2 x_4^2 \le 0.5^2$$

- **Non-negativity:**

$$x_i \ge 0, \ i = 1, 2, 3, 4$$

Model Building

Final step in building an optimization model:

- **Identify each restriction or limitation**
- **Express each restriction in terms of decision variables**

Model Building

Steps:

1. **Define a set of decision variables**

Hints:

Ask: **What can we manipulate control, or set the values? Or what decisions do we have to make?**

- **Minimum set of decision variables required**
- **A good model will not contain more variables than needed**

Model Building

Steps:

2. **Identify the objective function**

Hints:

Ask: **which criterion can you use to judge how good each option is? First state this in verbal form.**

Ask: **Whether you want to minimize or maximize the criterion?**

Model Building

Steps:

3. **Express the objective function in terms of the decision variables**

Hints:

- **Explore additivity & proportionality for a linear model**
- **Explore the following:**
  - * **Physical relationships**
  - * **Quantity-balance principle**
  - * **Logical or implied relationships**
  - * **Empirical modeling (such as regression etc.)**

Model Building

Steps:

4. Identify each restriction or limitation

Hints:

- **Ask:** **What make our choice of options limited?**
- **Source of constraints:**
  - * **Physical limitations**
  - * **Quantity-balance or systems dynamics**
  - * **Logical restrictions**
  - * **External Restrictions**
  - * **Management-imposed restrictions**

Model Building

Steps:

5. **Express each constraint in terms of the decision variables**

Hints:

- **Explore additivity & proportionality for a linear model**
- **Explore the following:**
  - * **Physical relationships**
  - * **Quantity-balance principle**
  - * **Logical or implied relationships**
  - * **Empirical modeling (such as regression etc.)**

Algebraic Optimization Model

$$\min f(\mathbf{x})$$
$$s.t.\ h_i(\mathbf{x}) = 0,\ i = 1, \ldots, l$$
$$g_j(\mathbf{x}) \le 0,\ j = 1, .., m$$
$$\mathbf{x} \in S \subseteq R^n$$

For example, for the Stock Investment Problem:

$\max f(\mathbf{x}) = 5000 \sum_{k=1}^{4} r_k x_k$ maximize expected return

$s.t.$

$\sum_{k=1}^{4} \sigma_k^2 x_k^2 \le 0.5^2$ Risk level maintained at desired level:

$\sum_{k=1}^{4} x_k \le 1$ Total investment is within budget:

$x_k \ge 0,\ k = 1,\ 2,\ 3,\ 4$ No negative investment (withdrawal)

Stock Investment Example:

Invest $5000 on 4 stocks whose expected returns and risks (standard deviation) per dollar invested are:

	Stock 1	Stock 2	Stock 3	Stock 4
Exp. Ret./$ invested	0.92	0.53	0.87	0.4
SD/$ invested	1	0.62	0.79	0.15

Maximize the expected total return, while maintaining the risk level below $0.50/1$ invested

Expected Return/$ invested	0.92	0.53	0.87	0.4
Standard deviation/$ invested	1	0.62	0.79	0.15

Our main objective is to maximize the expected total return,
while maintaining the combined risk level not exceeding 0.5 $ per $ invested

Solution:

Decision Variables:			Stock 1	Stock 2	Stock 3	Stock 4
Fraction of funds invested on stock i			0.317938	0.207481	0.46053	0.014051
Objective function:	0.808749	<= Expected Return in %				
Constraints:						
total invested:	1	=	1			
risk level	0.250001	<=	0.25			

Cellular Phone Company Example:

A company plans to add more transmission-reception cells to its existing network. 8 possible sites as shown below:

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Site 7	Site 8
sq. miles	5	3	7	2.5	9	4	7	8
cost ($M)	2	1.6	2.8	1	3.5	1.4	2.2	2.5

Investment Budget: **$7M**
If **site 5 is chosen**, then **site 2 must also be chosen**.
If **site 1 or 3 are chosen**, then **site 4 can not be** chosen
Which sites to choose to maximize the total area?
Assume no overlap of coverage from each project site.

Solution (Mobile Phone Co.):

Decision Variables:

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Site 7	Site 8
Site selectio	0	1	0	0	0	1	1	1

Objective Function: Area covered: 22

Constraints:

Budget:	7.7	<=	8	$M
Sites 5 & 2:	0	<=	1	
	F16	<=	C16	
Sites 1 & 4:	0	<=	1	
	B16	<=	1-E16	
Sites 3 & 4:	0	<=	1	
	D16	<=	1-E16	

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Exercise: Steel Production:				
Find a monthly production plan for steel 1				
and steel 2 to minimize production cost.				
	Steel 1		**Steel 2**	
	cost $/ton	mins/ton	cost $/ton	mins/ton
Mill 1	$ 10.00	20	$ 11.00	22
Mill 2	$ 12.00	24	$ 9.00	18
Mill 3	$ 14.00	28	$ 10.00	30

The company has a contract to supply 500 tons of steel 1 and 600 tons of steel 2 to its customer per week. Each of the three mills can operate an 8-hour per day for 5 days a week.

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Find a monthly production plan for steel 1 and steel 2					
to minimize production cost.					
	Steel 1		**Steel 2**		
	cost $/ton	mins/ton	cost $/ton	mins/ton	
Mill 1	$ 10.00	20	$ 11.00	22	
Mill 2	$ 12.00	24	$ 9.00	18	
Mill 3	$ 14.00	28	$ 10.00	30	
Decision Variables: Tons/month of steel i produced at Mill j					
	Steel 1	Steel 2			
Mill1	**500**	**0**			
Mill2	**0**	**600**			
Mill3	**0**	**0**			
Objective Function:		Total cost	**10400**		
Constraints:				D16 = sumproduct(B6	
Tons made	500	600		B19=sum(B12:B14); (	
	>=	>=			
Tons required	500	600			
Mins Avai		Mins Used			
12000	<=	10000		C24 = C6*B12+E6*C1	
12000	<=	10800		C25:C26 = copy of C2	
12000	<=	0			

Steel Example: LINDO Model

MIN $10x_{11}+12x_{12}+14x_{13}+11x_{21}+9x_{22}+10x_{23}$

ST

$20x_{11} + 22x_{21} < 12000$

$24x_{12} + 18x_{22} < 12000$

$28x_{13} + 30x_{23} < 12000$

$x_{11} + x_{12} + x_{13} > 500$

$x_{21} + x_{22} + x_{23} > 600$

Steel Example: LINGO Model

```
MODEL:
SETS:
MILLS/1..3/:MAXMINS;
STEELS/1..2/: DEMAND;
PAIR(MILLS, STEELS): COST, MINS, PROD;
ENDSETS
DATA:
COST=10, 11, 12, 9, 14, 10;
MINS= 20, 22, 24, 18, 28, 30; MAXMINS= 12000, 12000, 12000;
DEMAND= 500,600;
ENDDATA
MIN=@SUM(PAIR:COST*PROD);
@FOR(STEELS(J): @SUM(MILLS(I): PROD(I,J)) >
DEMAND(J));
@FOR(MILLS(I):@SUM(STEELS(J): MINS(I,J)*PROD(I,J)) <
MAXMINS(I));
END
```

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Example 2: Inscribing circles

Inscribe 2 non-overlapping circles in quadrilateral with vertices (0,0), (50,0), (40,20), and (20,30) and maximize the total area of the two circles.

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Algebraic Model:

Decision Variables:

Center of each circle: (x_i, y_i) = center of circle i, $i = 1, 2$

Radius of each circle: r_i = radius of circle i, $i = 1, 2$

Objective Function: Minimize waste

= area of rubber sheet - sum of areas of the two circles

≡ maximize sum of areas of the two circles = $\pi(r_1^2 + r_2^2)$

≡ maximize $r_1^2 + r_2^2$

Constraints:

Center of each circle must be in the quadrilateral: For each i, $i = 1, 2$:

$3x_i - 2y_i \geq 0$

$x_i + 2y_i \leq 80$

$2x_i + y_i \leq 100$

$y_i \geq 0$

It turns out that these constraints are redundant and can be removed.

The whole of each circle must be inside the quadrilateral: For each i, $i = 1, 2$:

$100 - 2x_i - y_i \geq r_i\sqrt{5}$

$80 - x_i - 2y_i \geq r_i\sqrt{5}$

$0 + 3x_i - 2y_i \geq r_i\sqrt{13}$

$y_i \geq r_i$

Nonoverlapping of the two circles:

$(x_1 - x_2)^2 + (y_1 - y_2)^2 \geq (r_1 + r_2)^2$

Nonnegativity: $r_i \geq 0$, $i = 1, 2$

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Solution (Inscribing Circles):											
Decision variables:											
Center 1	8.7865	4.702390737	Radius 1	4.7024							
Center 2	24.4867	13.10490379	Radius 2	13.105							
Objective function:											
Total area to be maximized:			609.001								
Constraints:					x	y					
Equations describing the quadrialateral:				Line 1	-3	2		<=	0		
				Line 2	1	2		<=	80		
				Line 3	2	1		<=	100		
				Line 4	0	-1		<=	0		
Center 1 must be inside:			**Line 1**	-16.955	<= 0	**Center 2 must be inside:**			Line 1	0	<= 0
			Line 2	18.191	<= 80				Line 2	0	<= 80
			Line 3	22.275	<= 100				Line 3	0	<= 100
			Line 4	-4.7024	<= 0				Line 4	0	<= 0
					x	y	radius r				
Equations describing distance from edge:				Line 1	-3	2	3.6056	<=	0		
				Line 2	1	2	2.2361	<=	80		
				Line 3	2	1	2.2361	<=	100		
				Line 4	0	-1	1	<=	0		
Circle 1 must be inside:			**Line 1**	0	<= 0	**Circle 2 must be inside:**			**Line 1**	0	<= 0
			Line 2	28.706	<= 80				**Line 2**	80	<= 80
			Line 3	32.79	<= 100				**Line 3**	91.382	<= 100
			Line 4	0	<= 0				**Line 4**	0	<= 0
Non-overlapping:		-1.81066E-07	>=	0							

This green part can be removed without affecting the correctness of the formulation.

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Circle Example: LINGO Model

```
MODEL:
max= r1^2+r2^2;
-3*x1+2*y1+13^0.5*r1<0;
x1+2*y1+5^0.5*r1<80;
2*x1+y1+5^0.5*r1<100; y1-r1>0;
-3*x2+2*y2+ 13^0.5*r2<0;
x2+2*y2+ 5^0.5*r2<80;
2*x2+y2+ 5^0.5*r2<100;
y2-r2>0;
(x1-x2)^2+(y1-y2)^2-(r1+r2)^2>0;
END
```

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Circle Example: LINGO Model 2

```
MODEL:
sets:
circles/1..2/: x, y, r; lines/1..4/: a1, a2, a0;
endsets
data:
a1=-3, 1, 2, 0; a2 = 2, 2, 1, -1; a0 = 0, 80,
100, 0;
enddata
max=@sum(circles(i):r(i)^2)*3.1416;
@for(circles(j):@for(lines(i):a1(i)*x(j)+a2(i)*y(j)
+(a1(i)^2+a2(i)^2)^0.5*r(j)<a0(i)));
(x(1)-x(2))^2+(y(1)-y(2))^2-(r(1)+r(2))^2>0;
END
```


Example 3: Police Scheduling

Number of polices required in each 6-hr period:

12am-6am	12
6am-12pm	8
12pm-6pm	6
6pm-12am	15

Police can be hired to work either 12 consecutives hours at $4/hr or 18 consecutive hours at $6 per hour beyond 12 hours of work.
Do police scheduling to meet daily requirements at minimum cost.

SOLUTION (Police Scheduling):

Decision Variables:

	# required	# of 12-hr	# of 18-hr
12am-6am	12	3	0
6am-12pm	8	0	5
12pm-6pm	6	1	0
6pm-12am	15	9	0
Total		13	5
Cost		48	84

Objective Function: Cost 1044

Constraints:

	12am	6am	12pm	6pm
3	1	1		
0		1	1	
1			1	1
9	1			1
0	1	1	1	
5		1	1	1
0	1		1	1
0	1	1		1
Scheduled	12	8	6	15
Required	12	8	6	15

Example 4: Alloy Production Planning

Requirements for steel production:

tensile strength at least 45,000 psi.

Steelco manufactures steel by combining two alloys.

	Alloy 1	Alloy 2
Cost per ton	$ 190.00	$ 200.00
% silicon	2	2.5
% nickel	1	1.5
% carbon	3	4
Tensile strength (psi	42000	50000

Assume tensile strenth of a mixture of the two alloys can be determined by averaging that are mixed together.

E.G., tensile strength of 1-ton mixture with 40% alloy 1& 60% alloy 2 .

= 0.4(42,000)+0.6(50,000)

Find production mix at minimum cost.

Tons of alloy i used to make 1 ton of steel:				0.625	0.375
Objective function:					
	Cost per ton of steel:		193.75		
Constraints:				min	max
% silicon achieved		2.1875		**1.8**	**2.5**
% nickel achieved		1.1875		**0.9**	**1.2**
% carbon achieved		3.375		**3.2**	**3.5**
tensile strength achieved		45000	>=	**45000**	
Sum of tons of alloys used:		1	=	**1**	

Example 5: Ambulance Location

The time in minutes it takes an ambulance to travel from one district to another is shown below. The population of each district in thousands is also shown. Find districts to locate 2 ambulances so as to maximize the number of people who lives within 2 minutes of an ambulance.

		To District							
		1	2	3	4	5	6	7	8
	1	0	3	4	6	8	9	8	10
From	2	3	0	5	4	8	6	12	9
District	3	4	5	0	2	2	3	5	7
	4	6	4	2	0	3	2	5	4
	5	8	8	2	3	0	2	2	4
	6	9	6	3	2	2	0	3	2
	7	8	12	5	5	2	3	0	2
	8	10	9	7	4	4	2	2	0
		8	***3***	***5***	***9***	***4***	***1***	***7***	***2***

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Solution (Ambulance

		District							
Decision Variables:		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Locate in district i?									
Objective Function:									
# of people within 2 minutes of ambulance:									
Constraints:									
		To District?							
		1	2	3	4	5	6	7	8
	1	1	0	0	0	0	0	0	0
2 mins	2	0	1	0	0	0	0	0	0
From	3	0	0	1	1	1	0	0	0
District	4	0	0	1	1	0	1	0	0
	5	0	0	1	0	1	1	1	0
	6	0	0	0	1	1	1	0	1
	7	0	0	0	0	1	0	1	1
	8	0	0	0	0	0	1	1	1
Number of ambulances		0	must equal		2				

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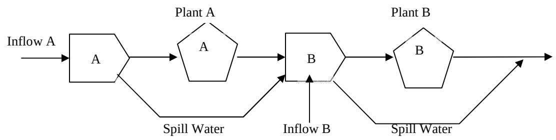
Solution (Ambulance

		District									
Decision Variables:		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*		
Locate in district i?	X_j	0	0	0	0	1	1	0	0		
District i covered?	Y_j	0	0	1	1	1	1	1	1		
Objective Function:											
# of people within 2 minutes of ambulance:					28						
Constraints:											
		To District?									
		1	2	3	4	5	6	7	8	Yi	Sum[a(ij)Xj)]
	1	1	0	0	0	0	0	0	0	0	0
2 mins	2	0	1	0	0	0	0	0	0	0	0
From	3	0	0	1	1	1	0	0	0	1	1
District	4	0	0	1	1	0	1	0	0	1	1
	5	0	0	1	0	1	1	1	0	1	2
	6	0	0	0	1	1	1	0	1	1	2
	7	0	0	0	0	1	0	1	1	1	1
	8	0	0	0	0	0	1	1	1	1	1
Number of ambulances		2	must equa		2						

Example **Hydropower Plant Scheduling**

A hydroelectric power system consists of two dams and their associated reservoirs and power plants on a river as shown below.



Assume flow rates in and out through the power plants are constant within each month. If the capacity of the reservoir is exceeded, the excess water runs down the spillway and bypasses the power plant. A consequence of these assumptions is that the maximum and minimum water-level constraints need to be satisfied only at the end of the month. Other operating characteristics of the reservoirs and power plants are given in the table in the next slide, which all quantities measuring water are in units of cubic acre-feet (KAF) and power is measured in megawatt-hours (MWH).

Power can be sold at $5.00 per MKH for up to 50,000 MWH each month, and excess power above that figure can be sold for $3.50 per MWH. Formulate a mathematical programming model to find an optimal operation strategy for the reservoirs and power plants during March and April.

Example: Power Plant Scheduling					
Data:					
	A	B	Units		
Reservoir					
Max cap	2000	1500	KAF		
Min Cap	1200	800	KAF		
Inflow					
March	200	40	KAF		
April	130	15	KAF		
March 1 level	1900	850	KAF		
P_plant cap	60000	35000	MWH		
Water-Power	400	200	MWH/KAF		
			Note		
Price ($/KWH)	5	Range 1	0	<= MWH <=	50000
	3.5	Range 2	50000	<= MWH	

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Decision Variables										
	A		B							
	March	April	March	April						
Release (KAF/m)	150	87.5	0	0						
End-of-m Level (KAF)	1950	1992.5	1040	1142.5						
Spilled (KAF)	0	0	0	0						
Power (MWH/m)	60000	35000	0	0	<= Power conversion					
							Note: Color codes of values:			
							Red = data			
Power sold in range 1	50000	35000					Yellow background = Decision Variables			
Power sold in range 2	10000	0					Light blue background = computed quantitie			
							Pink background = objective value			
Objective Function	460000		= B17*sum(B30:C30)+B18*sum(B31:C31) or							
			=SUMPRODUCT(B17:B18,B30:B31)+SUMPRODUCT(B17:B18,C30:C31)							
Constraints:										
	A		B							
	March	April	March	April						
Reservoir cap: Max	2000	2000	1500	1500						
Reservoir cap: Min	1200	1200	800	800						
Power	March	April								
Total produced (MWH)	60000	35000								
Max (MWH)	60000	35000								
Water balance:	A		B							
	March	April	March	April						
End-of-month level	1950	1992.5	1040	1142.5						
Ini+Inf-R-S	1950	1992.5	1040	1142.5						
Logical constraints										
Power sold in range 1 <-= 50000, (see implementation in Solver Dialog box)										
	March	April								
Power in range 1+ range2	60000	35000								
= Total power produced										
Nonegativity:	See implementation in Solver Dialog box									